Lecture 8

2021/2022

Microwave Devices and Circuits for Radiocommunications

2021/2022

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course + laboratory)**
- Lectures- **associate professor Radu Damian**
  - Monday 11-13, **P8**, ~~Microsoft Teams~~
  - E – 50% final grade, room?
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L2: 28.02.2022 (t2 and t3 not announced)
    - 3p=+0.5p
  - **all materials/equipments authorized**

2021/2022

- Laboratory – **associate professor Radu Damian**
  - Thursday 16-18, II.13
  - Friday **8**-12, II.13, odd week
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + personal results
  - P – 25% final grade
    - ADS, 3 sessions (-1? 25.02.2022)
    - personal homework

Materials

- RF-OPTO
  - http://rf-opto.etti.tuiasi.ro
- David Pozar, "Microwave Engineering", Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by ~~email~~/online exam
  - used at lectures/laboratory

Profile photo

- Profile photo – online “exam”

Examene online: 2020/2021

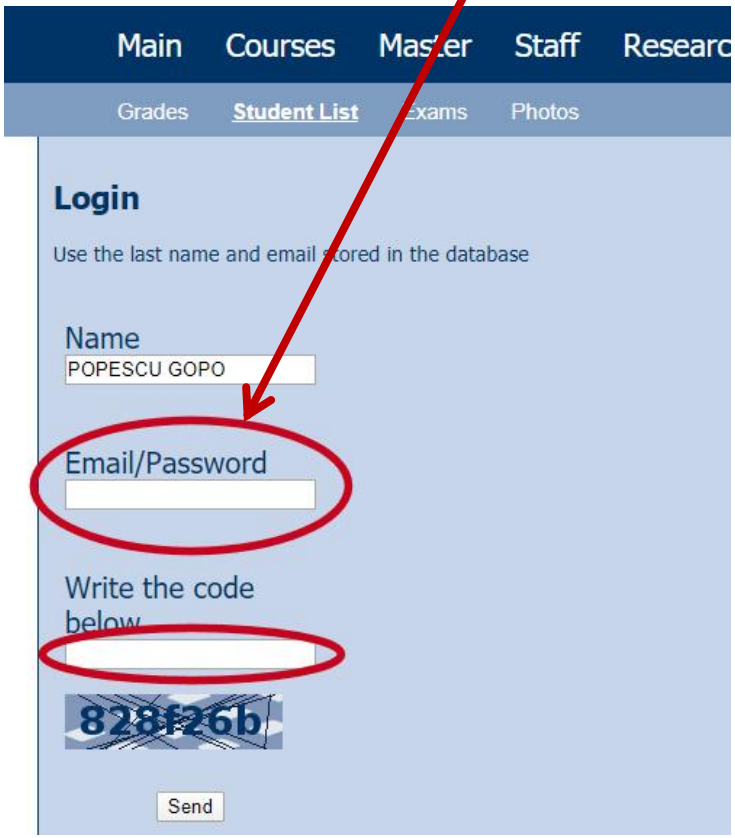
Disciplina: MDC (Microwave Devices and Circuits (Engleza))

Pas 3

Nr.	Titlu	Start	Stop	Text
1	Profile photos	03/03/2021; 10:00	08/04/2021; 08:00	Online "exam" created f ..
2	Mini Test 1 (lecture 2)	03/03/2021; 15:35	03/03/2021; 15:50	The current test consis ..

Online

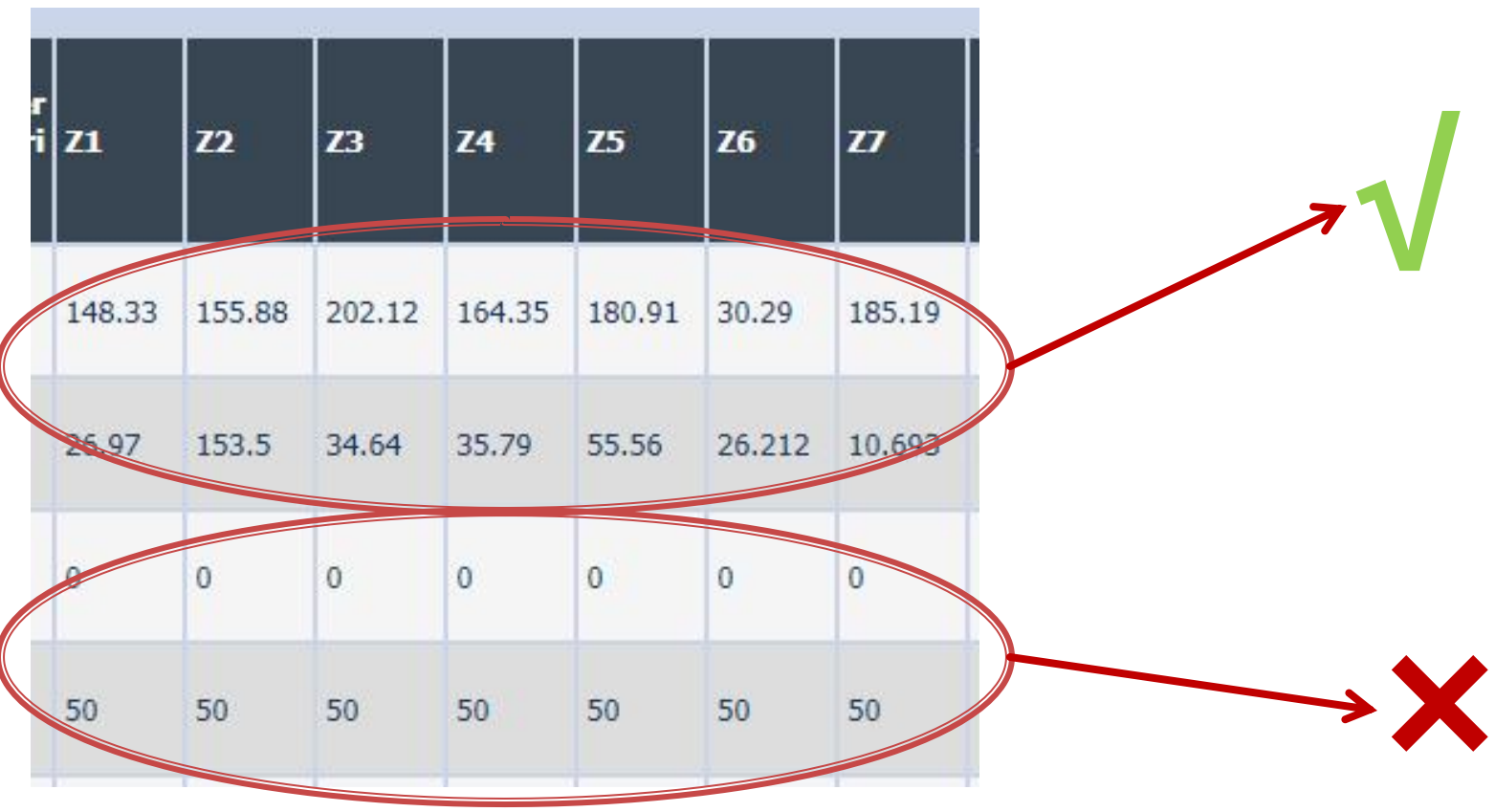
- access to **online exams** requires the **password** received by email



Online results submission

- many numerical values

Z1	Z2	Z3	Z4	Z5	Z6	Z7
148.33	155.88	202.12	164.35	180.91	30.29	185.19
26.97	153.5	34.64	35.79	55.56	26.212	10.693
0	0	0	0	0	0	0
50	50	50	50	50	50	50



Online results submission

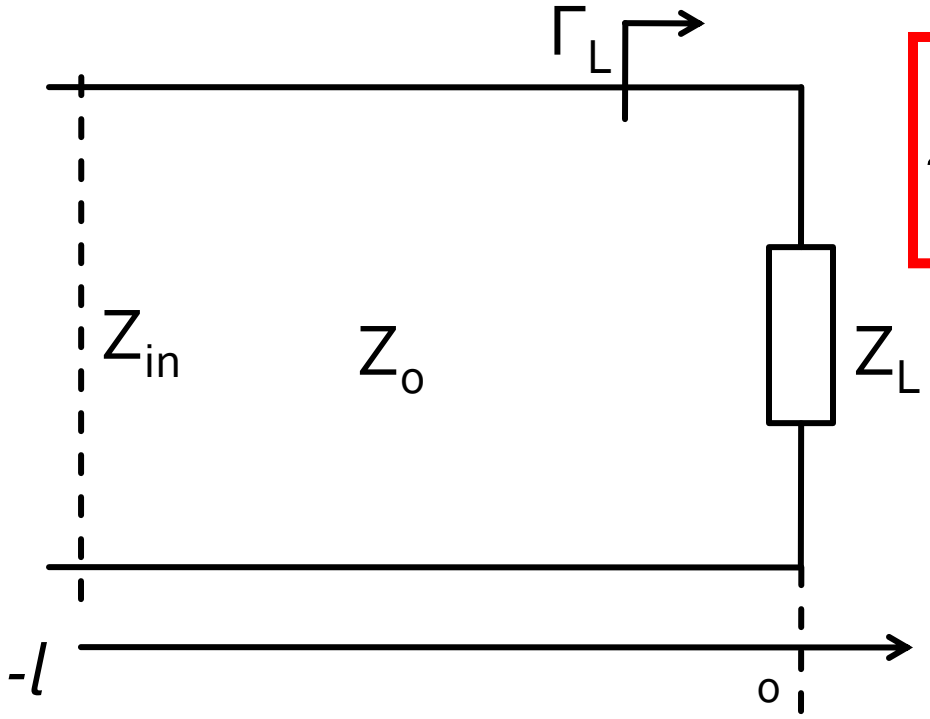
Grade = Quality of the work +

+ Quality of the submission

Important

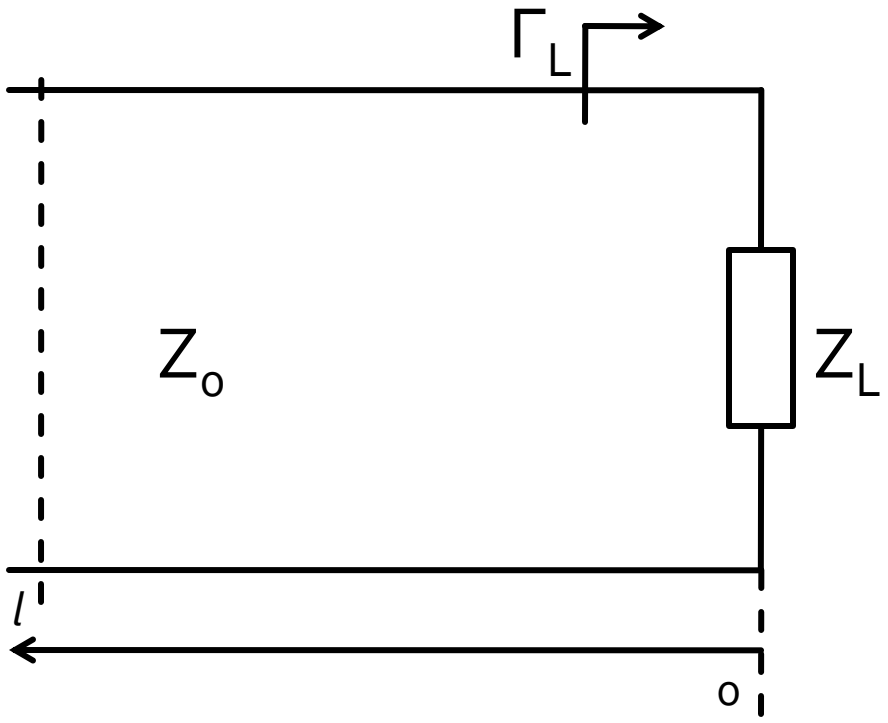
The lossless line

- input impedance of a length ***l*** of transmission line with characteristic impedance Z_o, loaded with an arbitrary impedance Z_L



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line



$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \qquad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\cdot Z_0$$

- voltage reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- Z_o real

The lossless line

$$V(z) = V_0^+ \cdot \left(e^{-j\cdot\beta\cdot z} + \Gamma \cdot e^{j\cdot\beta\cdot z} \right) \qquad I(z) = \frac{V_0^+}{Z_0} \cdot \left(e^{-j\cdot\beta\cdot z} - \Gamma \cdot e^{j\cdot\beta\cdot z} \right)$$

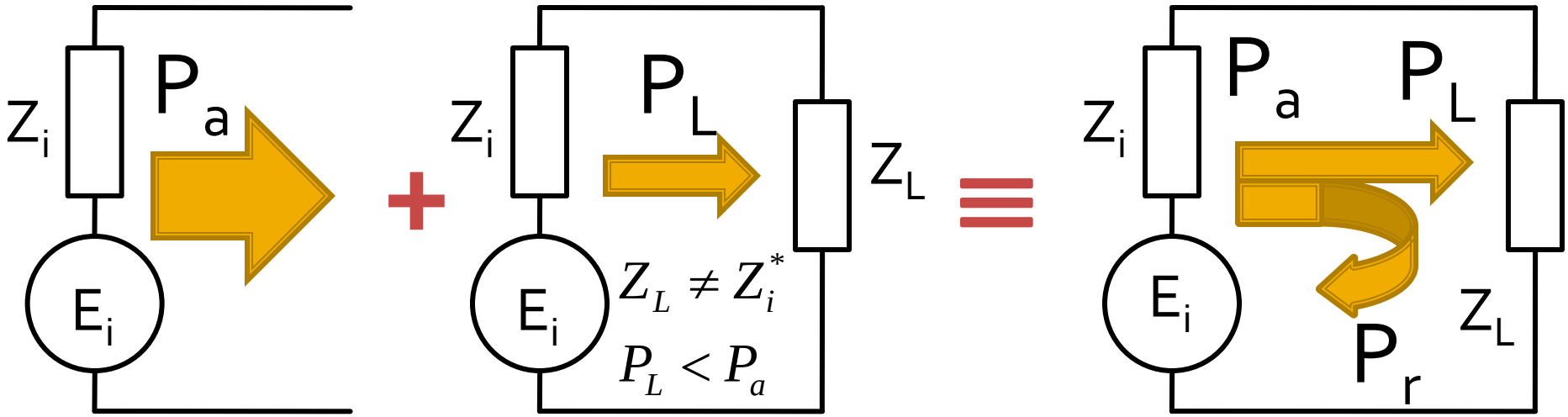
- time-average Power flow along the line

$$P_{avg} = \frac{1}{2} \cdot \mathrm{Re}\left\{ V(z) \cdot I(z)^* \right\} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \mathrm{Re}\left\{ 1 - \Gamma^* \cdot \underbrace{e^{-2j\cdot\beta\cdot z} + \Gamma \cdot e^{2j\cdot\beta\cdot z}}_{\left(z - z^*\right) = \mathrm{Im}} - \left|\Gamma\right|^2 \right\}$$

$$P_{avg} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \left(1 - \left|\Gamma\right|^2 \right)$$

- Total power delivered to the load = Incident power – "Reflected" power
- Return "Loss" [dB] $\qquad \mathrm{RL} = -20 \cdot \log\left|\Gamma\right| \quad [\mathrm{dB}]$

Reflection and power / Model



- The source has the ability to sent to the load a certain maximum power (available power) P_a
- For a particular load the power sent to the load is less than the maximum (mismatch) $P_L < P_a$
- The phenomenon is **"as if"** (model) some of the power is reflected $P_r = P_a - P_L$
- The power is a **scalar !**

Matching , from the point of view of power transmission

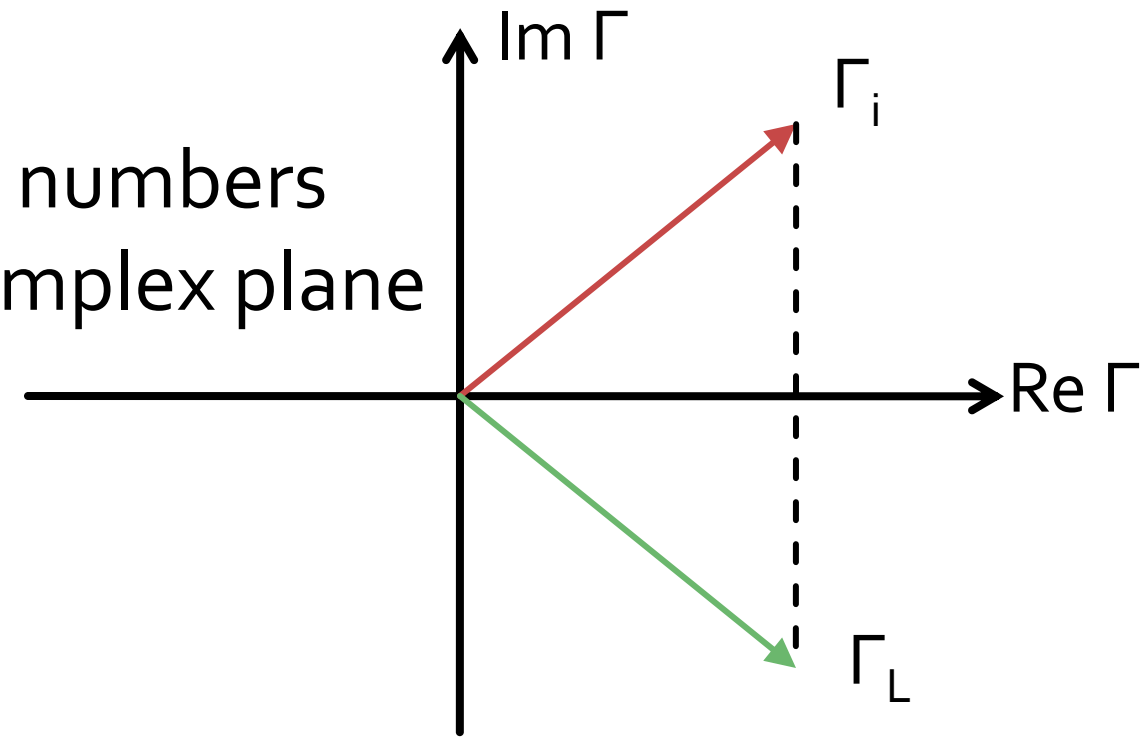
$$Z_L = Z_i^*$$

If we choose a real Zo

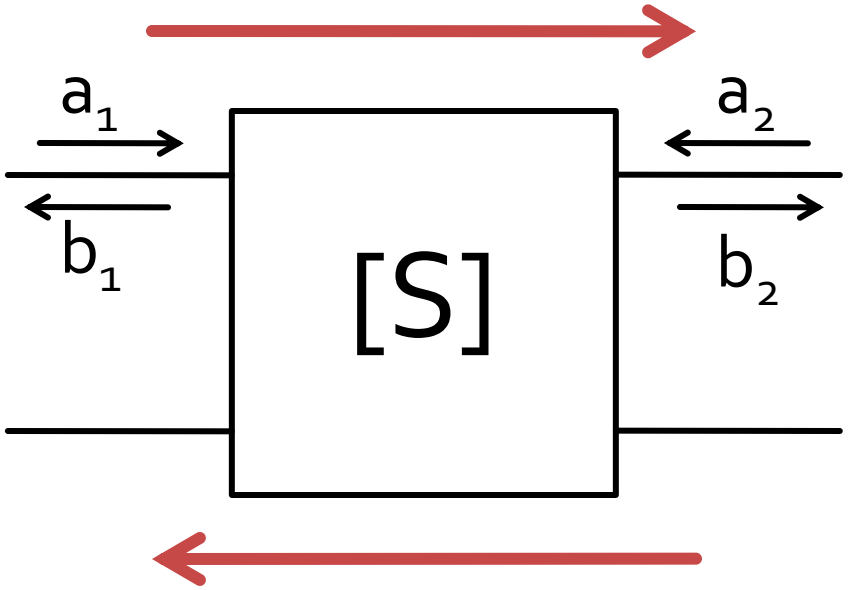
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

- complex numbers
- in the complex plane



Scattering matrix – S

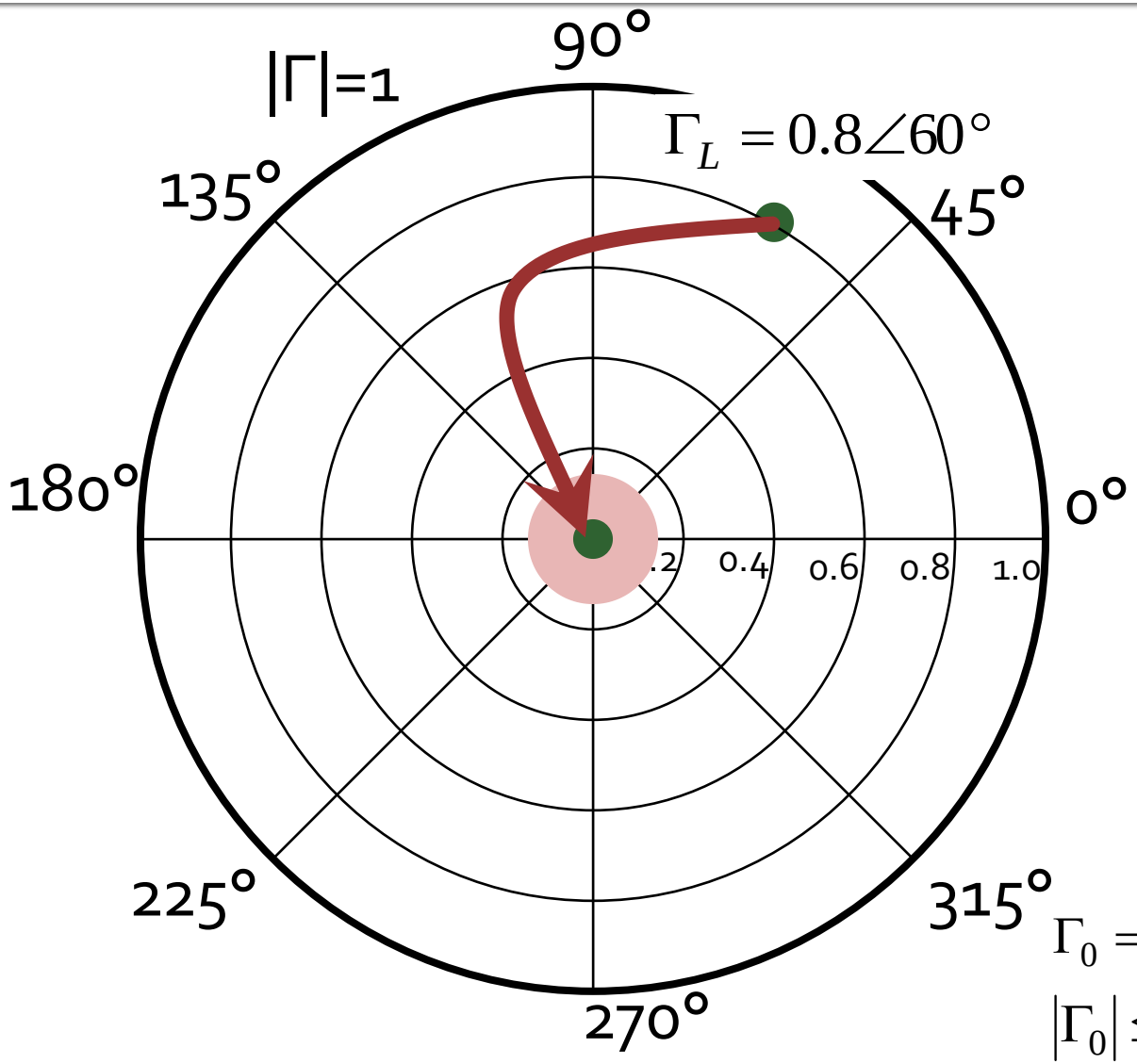


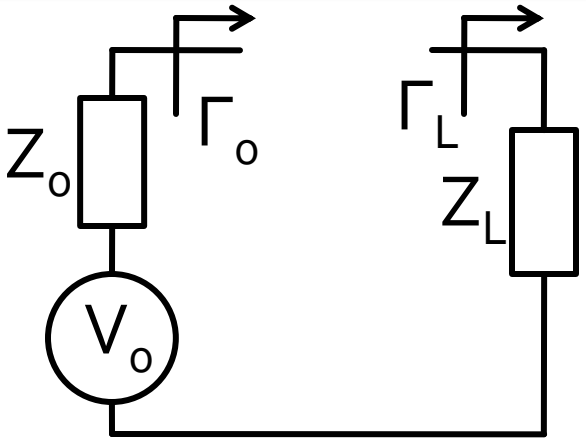
$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ \ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

Impedance matching



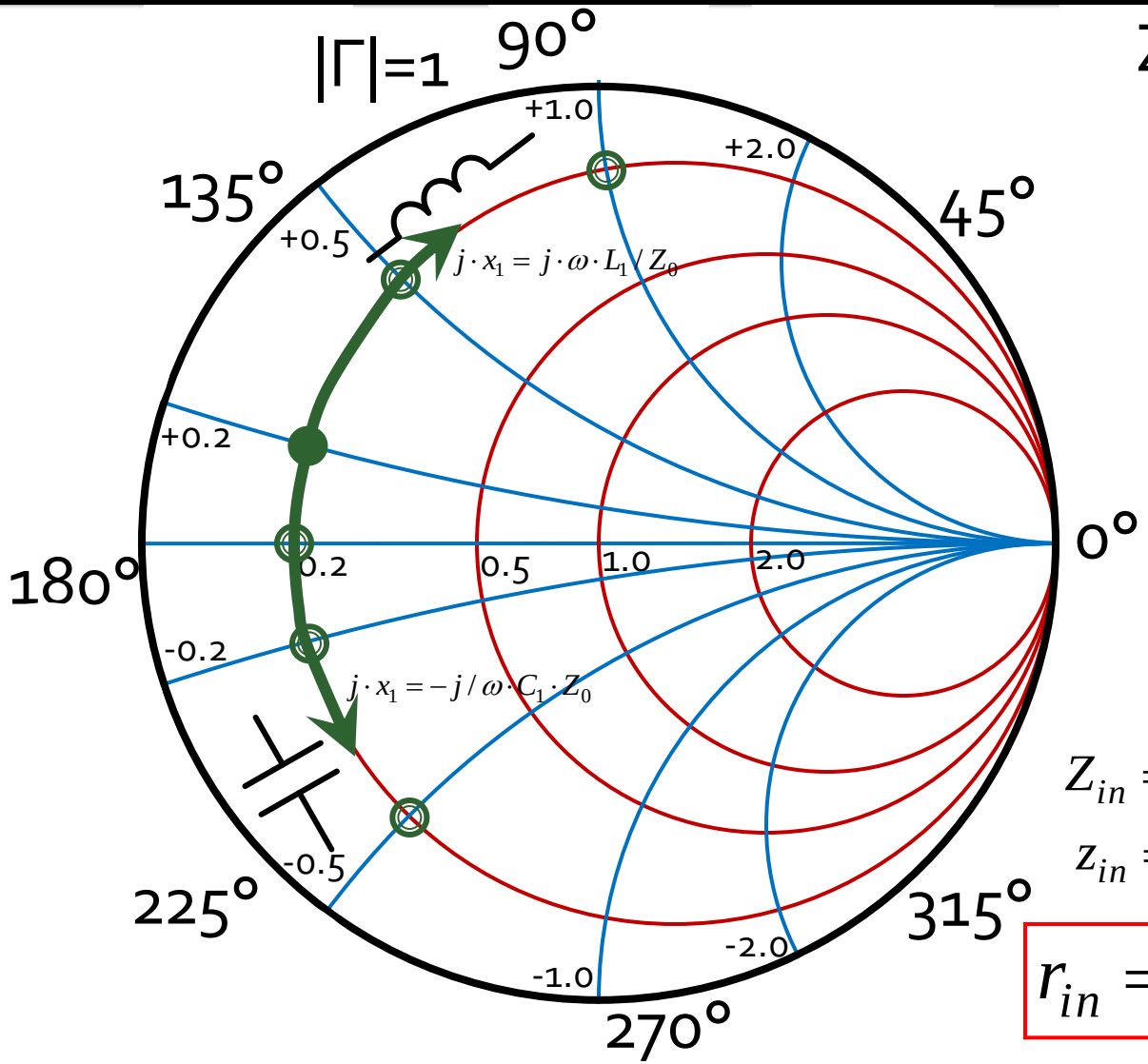


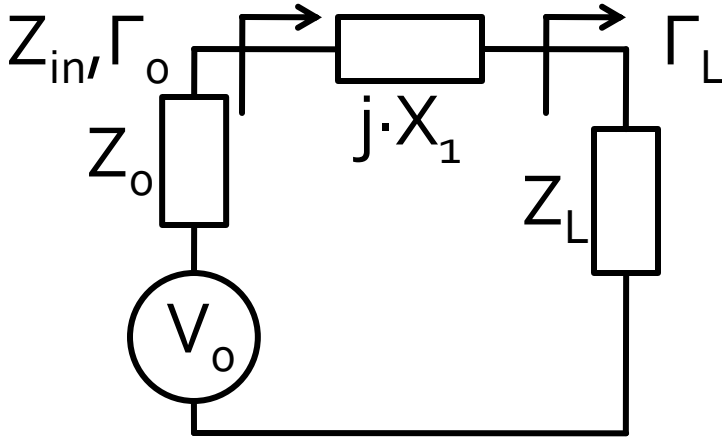
How?

$\Gamma_0 = 0$ perfect match

$|\Gamma_0| \le \Gamma_m$ "good enough" match

The Smith Chart, series reactance





$$Z_0 = 50\Omega$$

$$Z_L = R_L + j \cdot X_L = 10\Omega + j \cdot 10$$

$$z_L = r_L + j \cdot x_L = 0.2 + j \cdot 0.2$$

$$\Gamma_L = 0.678 \angle 156.5°$$

$$Z_{in} = Z_L + j \cdot X_1 = R_L + j \cdot (X_L + X_1$$

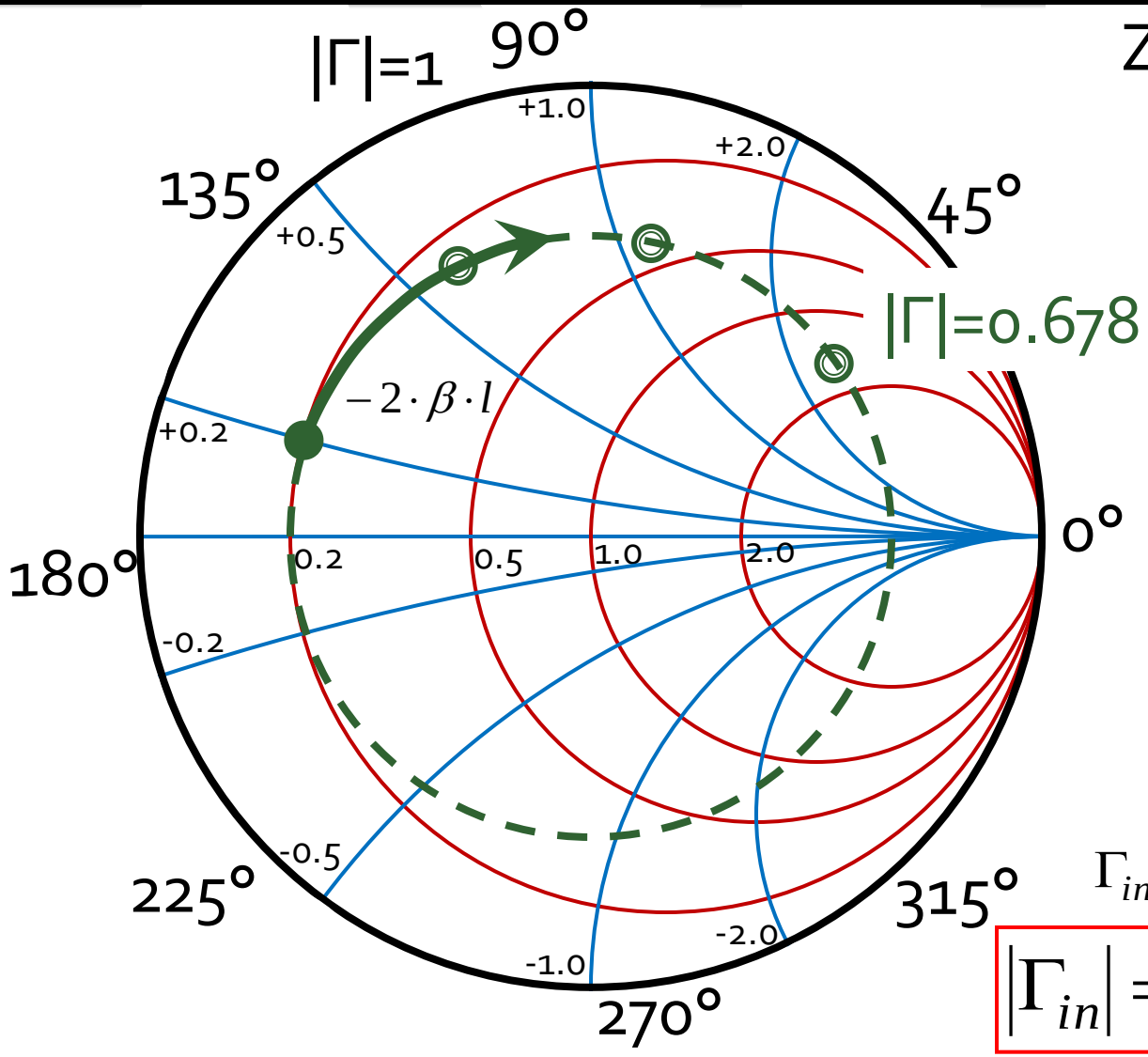
$$z_{in} = r_L + j \cdot (x_L + x_1)$$

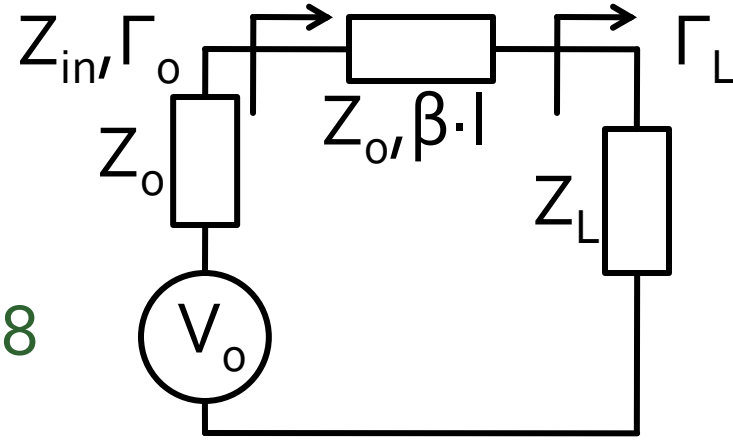
$$r_{in} = r_L$$

$$j \cdot x_1 = j \cdot \omega \cdot L_1 / Z_0 > 0$$

$$j \cdot x_1 = -j / \omega \cdot C_1 \cdot Z_0 < 0$$

The Smith Chart, series transmission line, Z_0





$$Z_0 = 50\Omega$$

$$Z_L = R_L + j\cdot X_L = 10\Omega + j\cdot 10$$

$$z_L = r_L + j\cdot x_L = 0.2 + j\cdot 0.2$$

$$\Gamma_L = 0.678\angle 156.5°$$

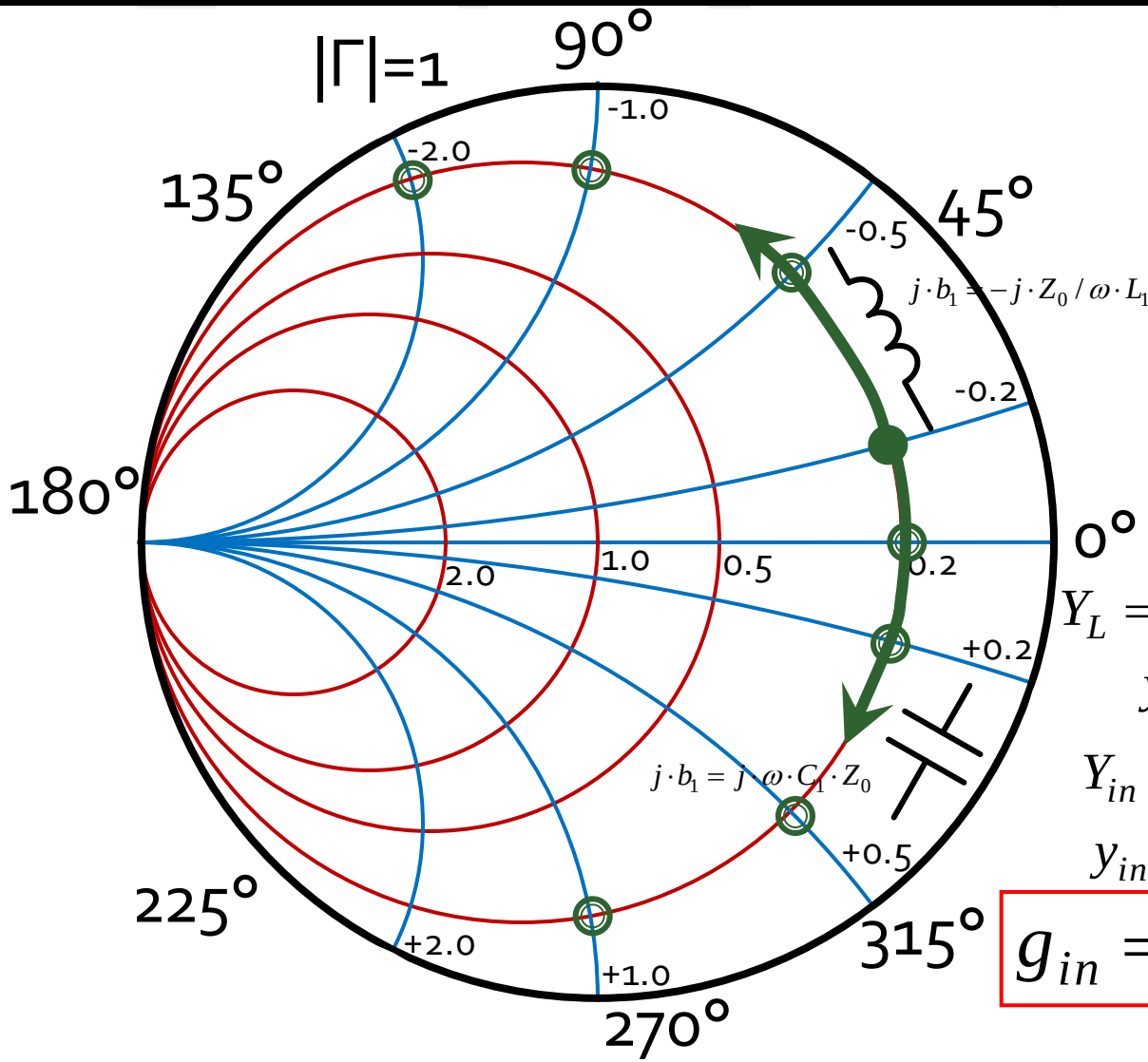
$$Z_{in} = Z_0\cdot\frac{1+\Gamma_L\cdot e^{-2j\cdot\beta\cdot l}}{1-\Gamma_L\cdot e^{-2j\cdot\beta\cdot l}}$$

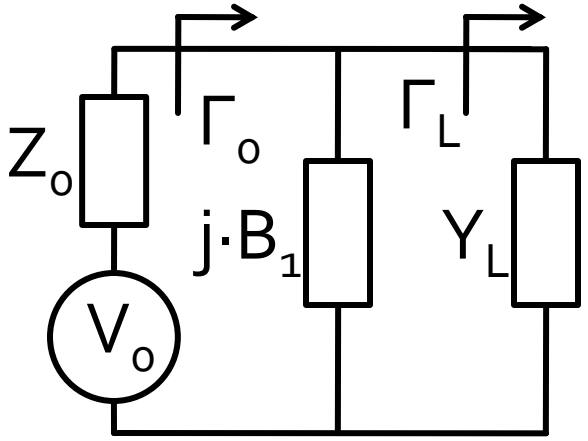
$$\Gamma_{in} = \Gamma_L\cdot e^{-2j\cdot\beta\cdot l}$$

$$\left|\Gamma_{in}\right| = \left|\Gamma_L\right|$$

$$\arg(\Gamma_{in}) = \arg(\Gamma_L) - 2\cdot\beta\cdot$$

The Smith Chart, shunt susceptance





$$Z_0 = 50\Omega, Y_0 = 0.02\,S$$

$$\Gamma_L = 0.678 \angle 23.5°$$

$$Y_L = G_L + j \cdot B_L = 0.004\,S + j \cdot 0.004$$

$$y_L = g_L + j \cdot b_L = 0.2 - j \cdot 0.2$$

$$Y_{in} = Y_L + j \cdot B_1 = G_L + j \cdot (B_L + B_1)$$

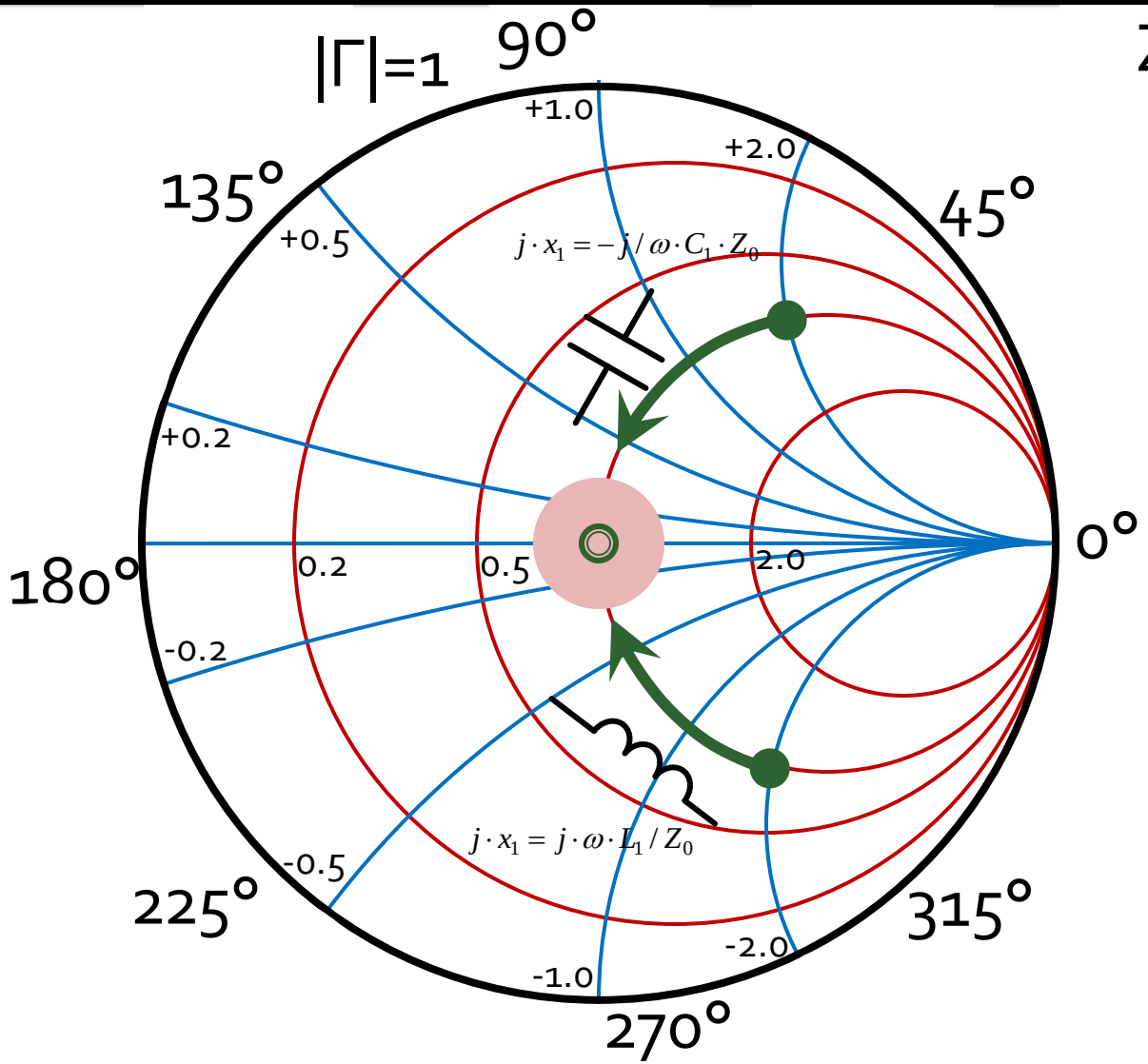
$$y_{in} = g_L + j \cdot (b_L + b_1)$$

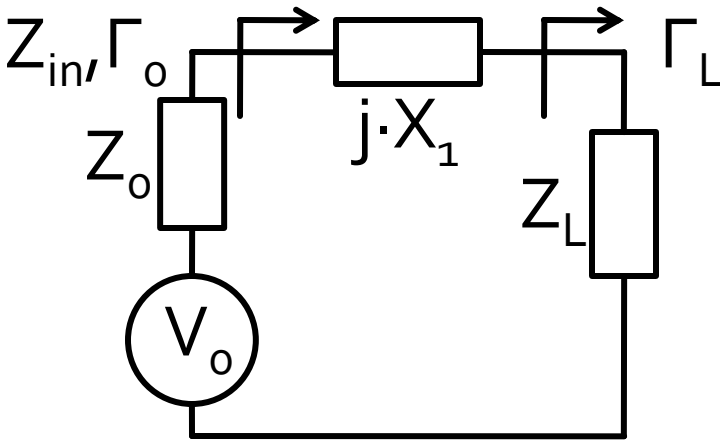
$$g_{in} = g_L$$

$$j \cdot b_1 = j \cdot \omega \cdot C_1 \cdot Z_0 > 0$$

$$j \cdot b_1 = -j \cdot Z_0 / \omega \cdot L_1 < 0$$

Matching, series reactance





$$z_L = r_L + j \cdot x_L$$

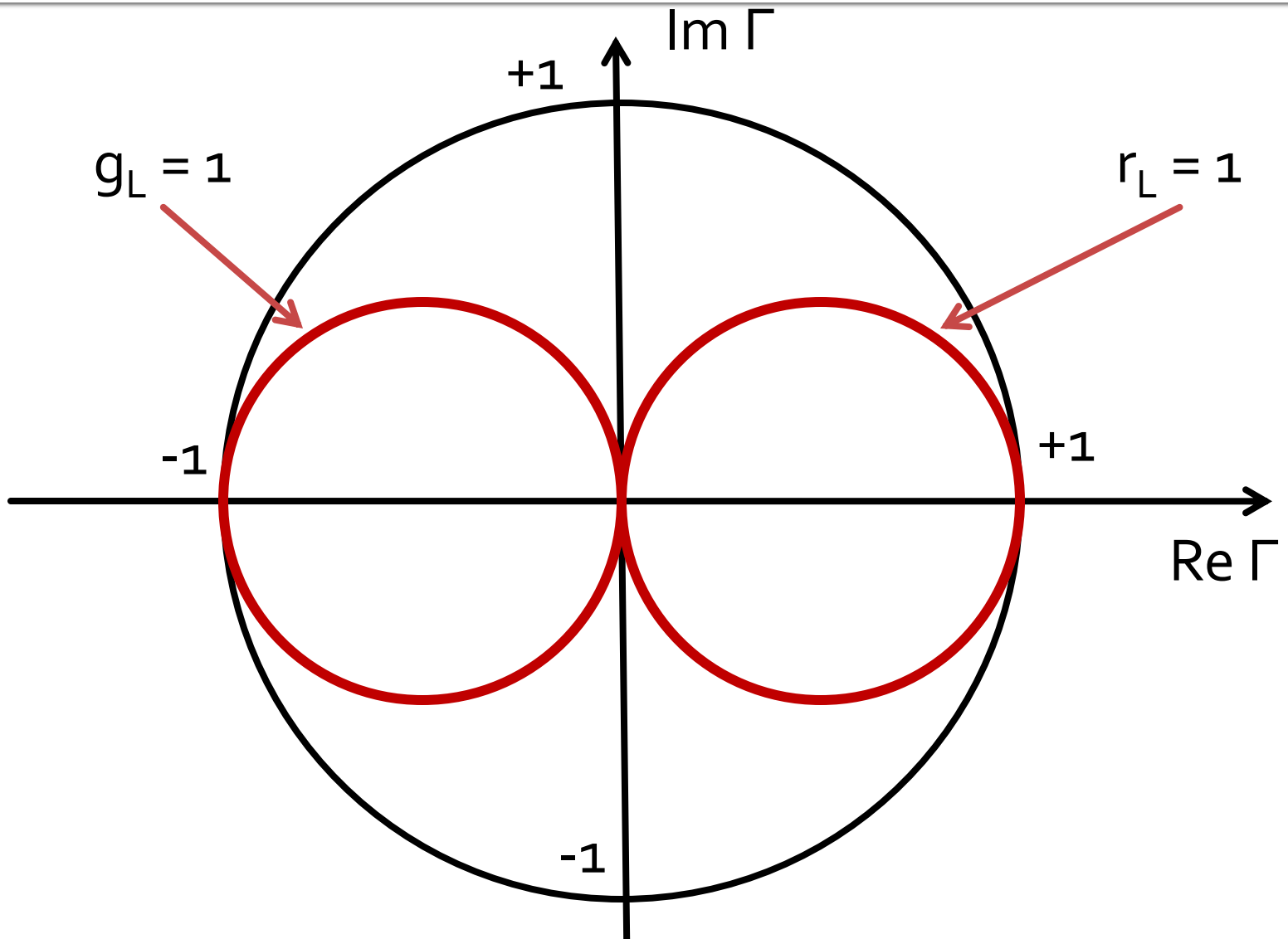
$$z_{in} = r_L + j \cdot (x_L + x_1)$$

$$r_{in} = r_L$$

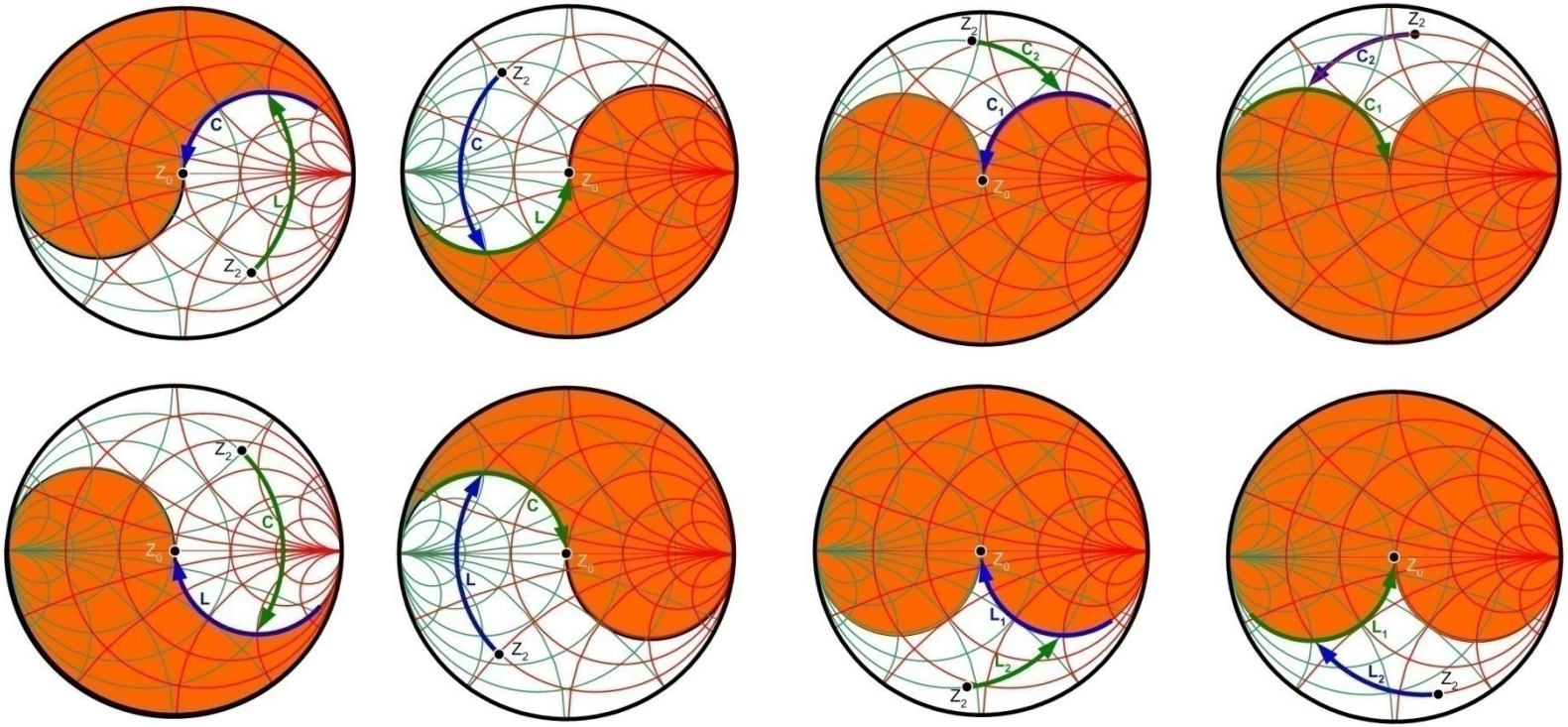
- Match can be obtained **if and only if** $r_L = 1$
- we compensate the reactive part of the load

$$j \cdot x_1 = -j \cdot x_L$$

Smith chart, r=1 and g=1



Matching with 2 reactive elements (L Networks)

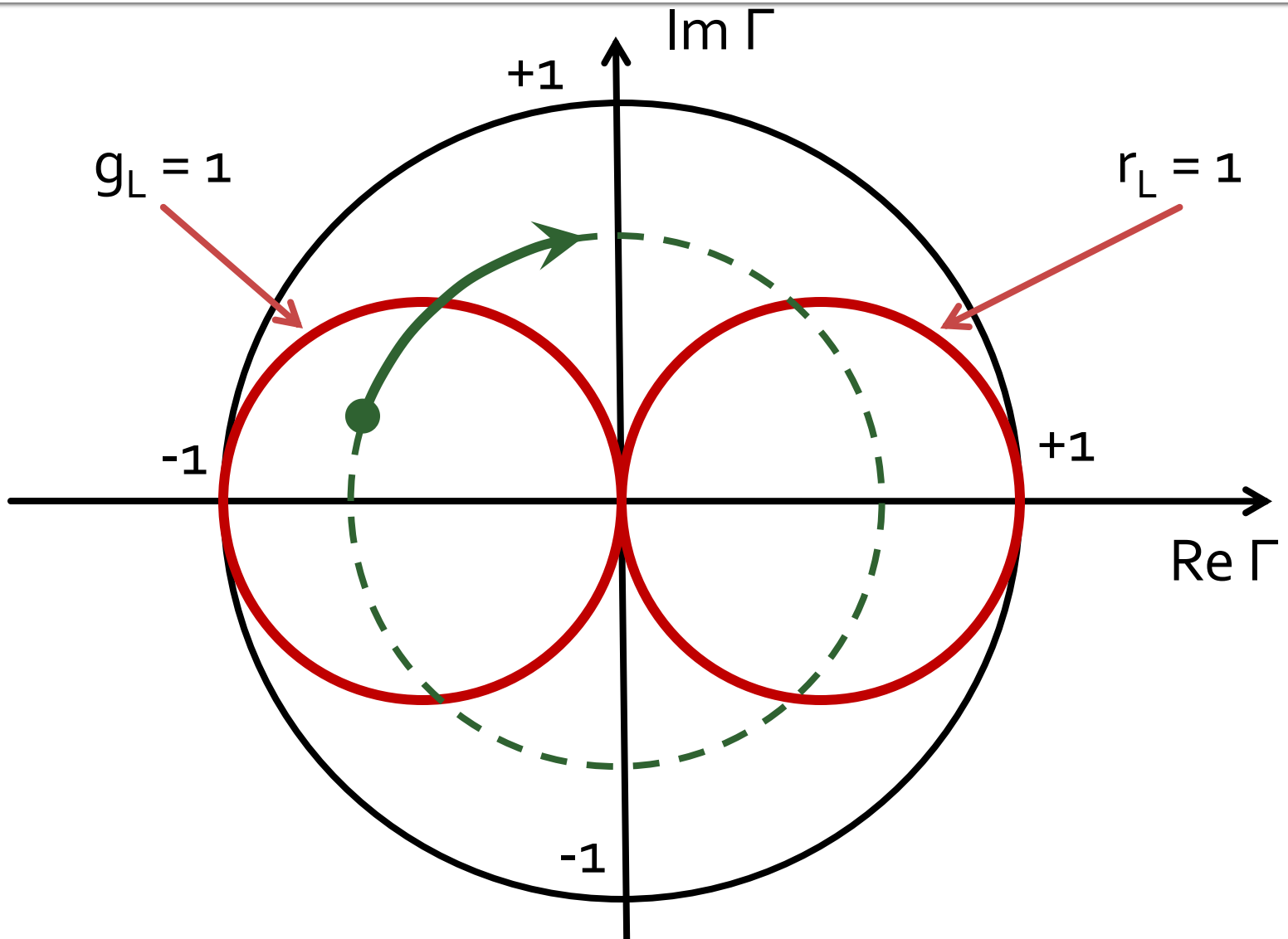


Forbidden area for current network

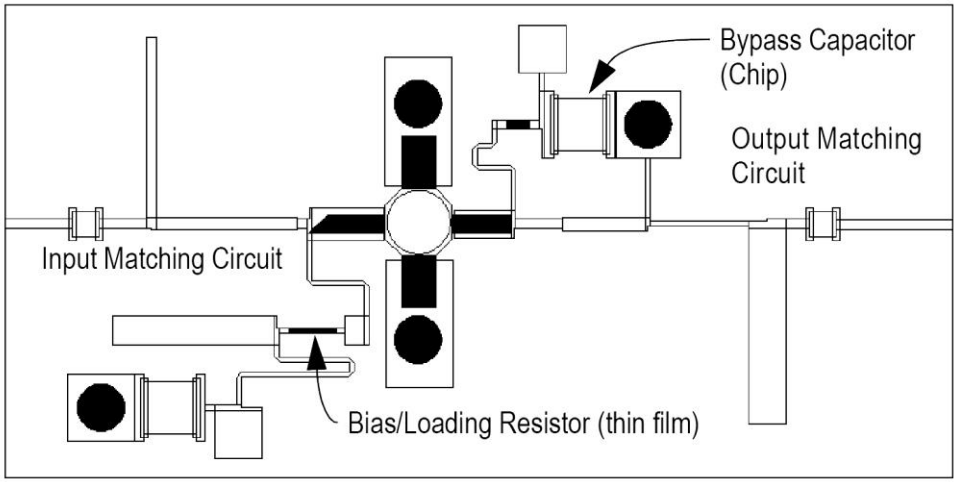
Impedance Matching

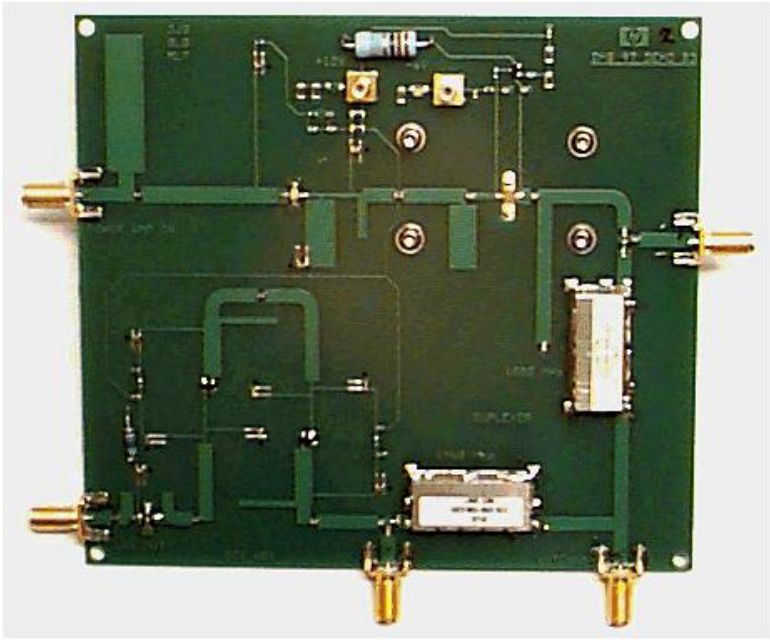
Impedance Matching with Stubs

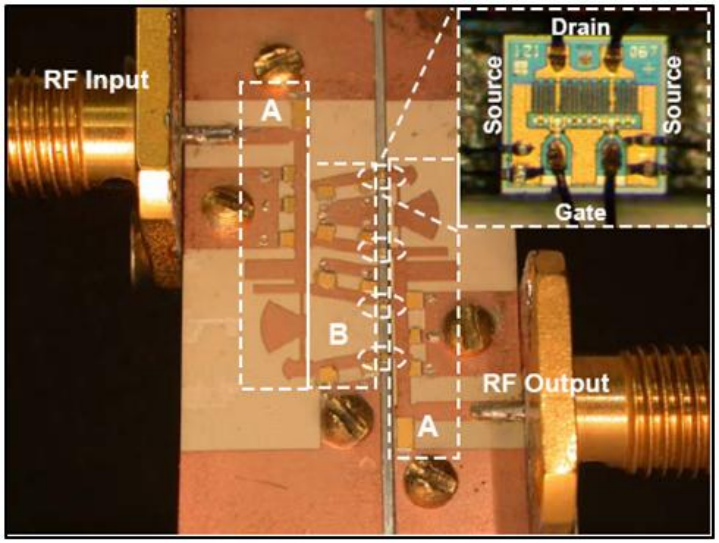
Smith chart, r=1 and g=1

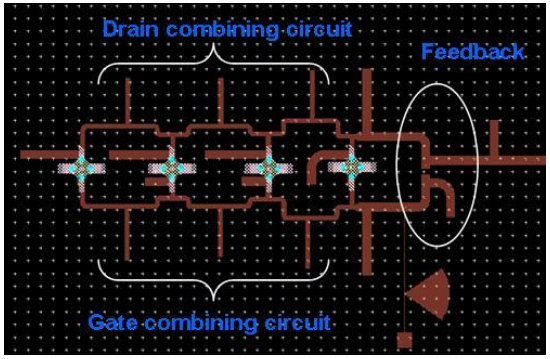


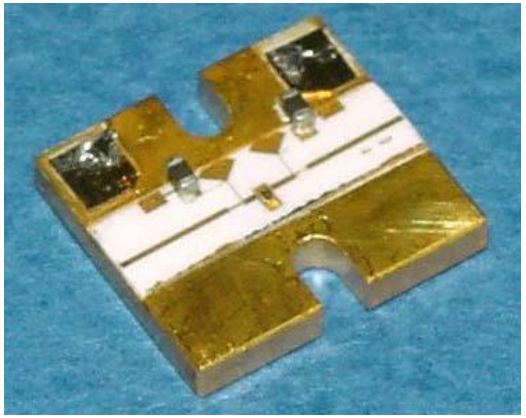
Impedance Matching with Stubs









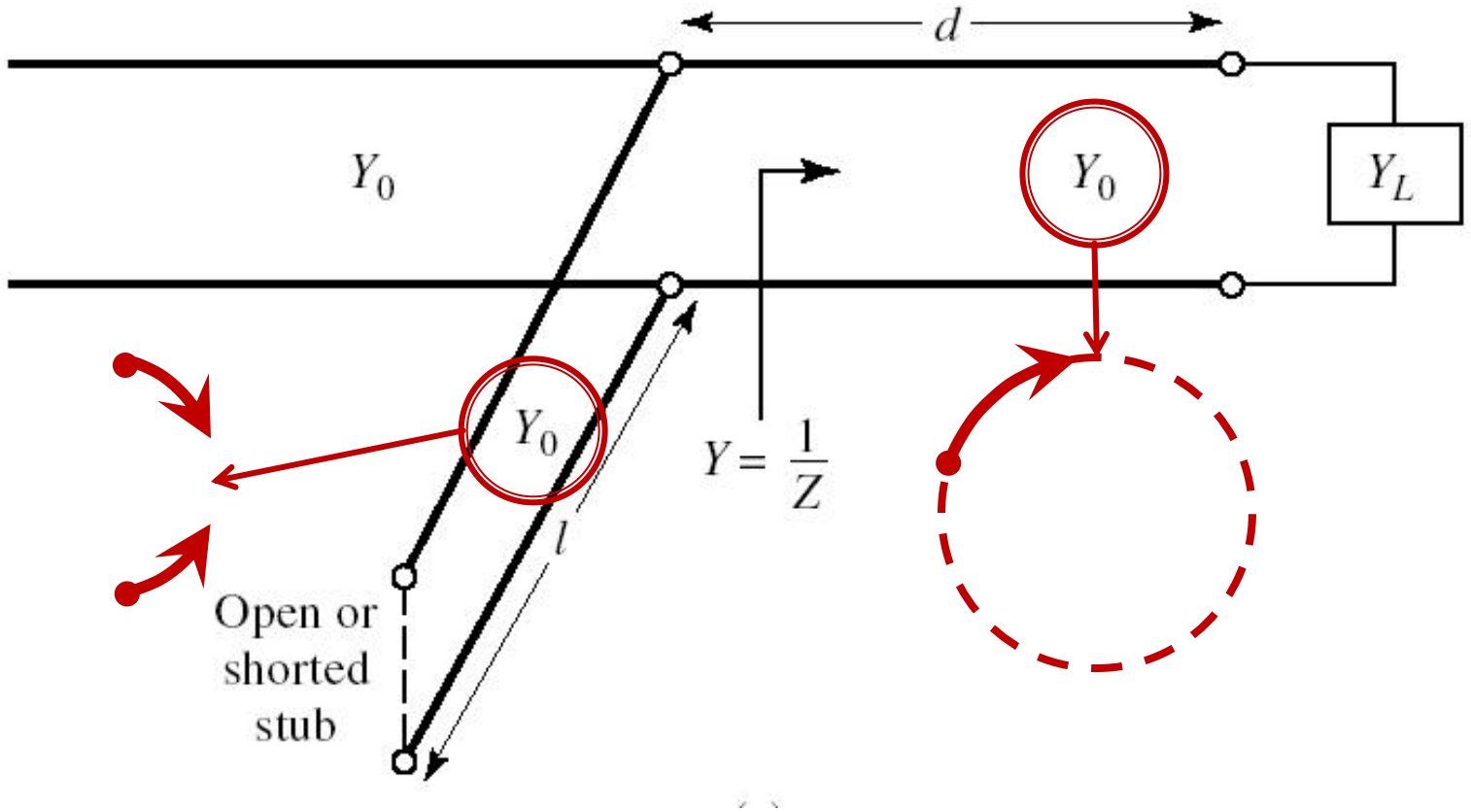


Analytical solutions

Exam / Project

Case 1, Shunt Stub

- Shunt Stub



Analytical solution, usage

$$\cos(\varphi + 2\theta) = -|\Gamma_S| \qquad \theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.593 \angle 46.85^\circ$$

$$|\Gamma_S| = 0.593; \quad \varphi = 46.85^\circ \qquad \cos(\varphi + 2\theta) = -0.593 \Rightarrow \quad (\varphi + 2\theta) = \pm 126.35^\circ$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

$$(46.85^\circ + 2\theta) = +126.35^\circ \qquad \theta = +39.7^\circ \qquad \mathrm{Im}\, y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.472$$

$$\theta_{sp} = \tan^{-1}(\mathrm{Im}\, y_S) = -55.8^\circ (+180^\circ) \rightarrow \theta_{sp} = 124.2^\circ$$

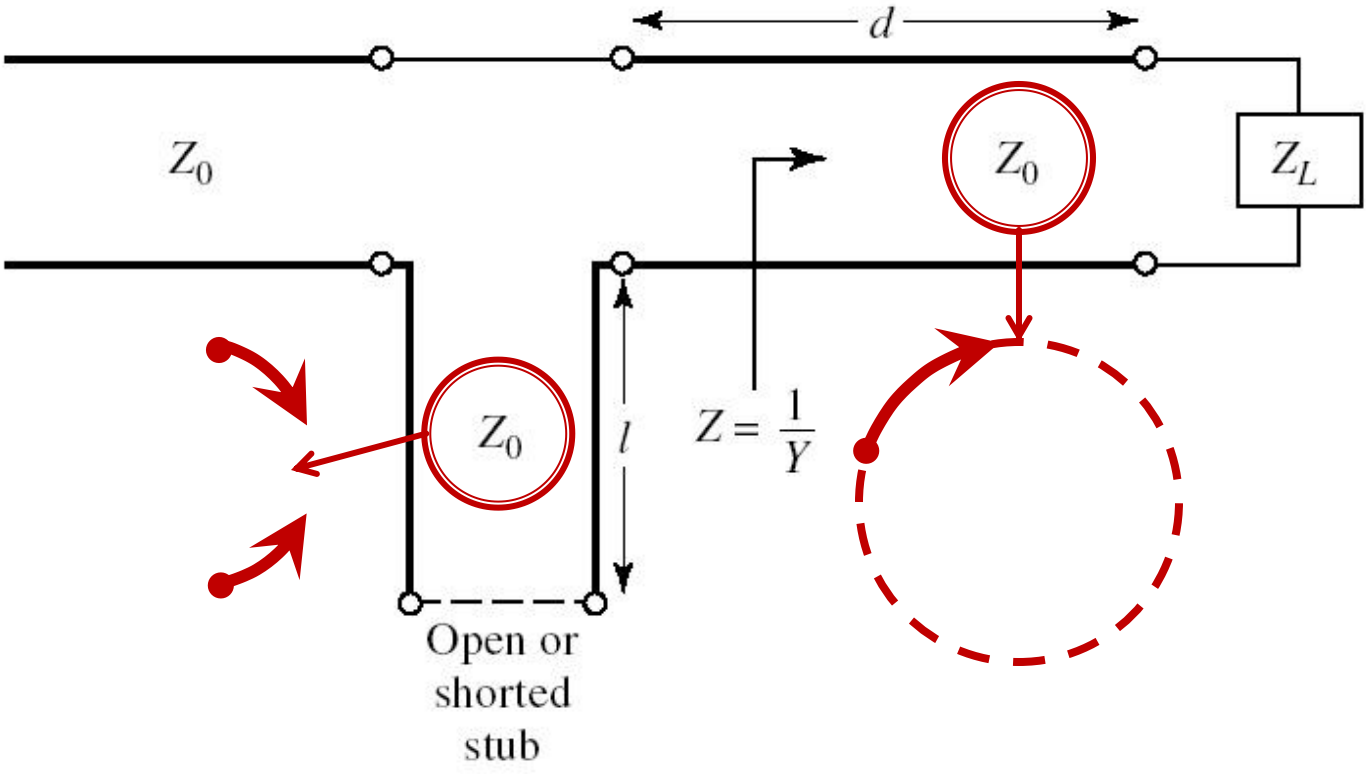
  - **"-" solution**

$$(46.85^\circ + 2\theta) = -126.35^\circ \qquad \theta = -86.6^\circ (+180^\circ) \rightarrow \theta = 93.4^\circ$$

$$\mathrm{Im}\, y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.472 \qquad \theta_{sp} = \tan^{-1}(\mathrm{Im}\, y_S) = 55.8^\circ$$

Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



Analytical solution, usage

$$\cos(\varphi + 2\theta) = |\Gamma_S|$$

$$\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.555 \angle -29.92^\circ$$

$$|\Gamma_S| = 0.555; \quad \varphi = -29.92^\circ \qquad \cos(\varphi + 2\theta) = 0.555 \Rightarrow (\varphi + 2\theta) = \pm 56.28^\circ$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation
  - **"+" solution**

$$(-29.92^\circ + 2\theta) = +56.28^\circ \qquad \theta = 43.1^\circ \qquad \mathrm{Im} z_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.335$$

$$\theta_{ss} = -\cot^{-1}(\mathrm{Im} z_S) = -36.8^\circ (+180^\circ) \rightarrow \theta_{ss} = 143.2^\circ$$

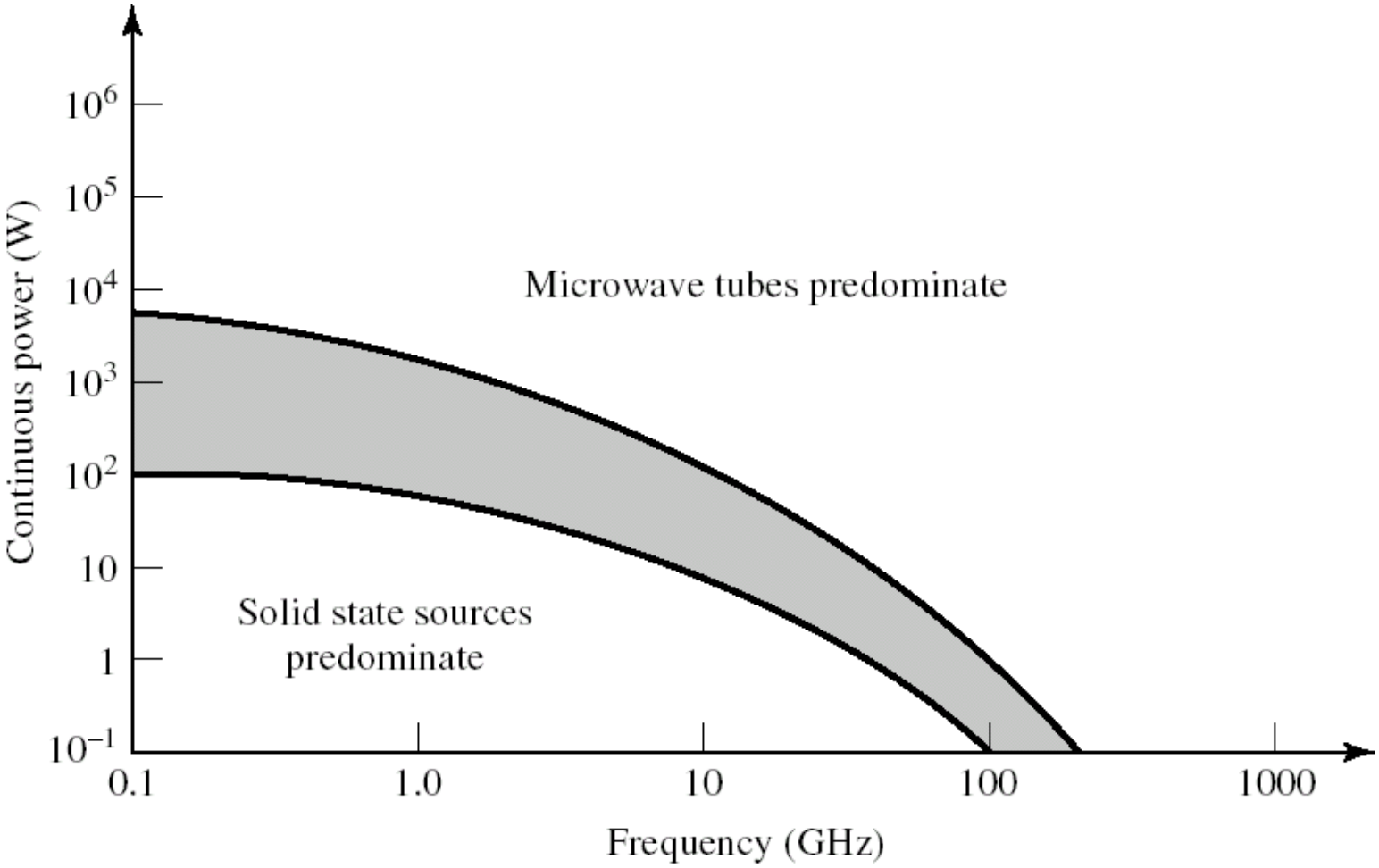
  - **"-" solution**

$$(-29.92^\circ + 2\theta) = -56.28^\circ \qquad \theta = -13.2^\circ (+180^\circ) \rightarrow \theta = 166.8^\circ$$

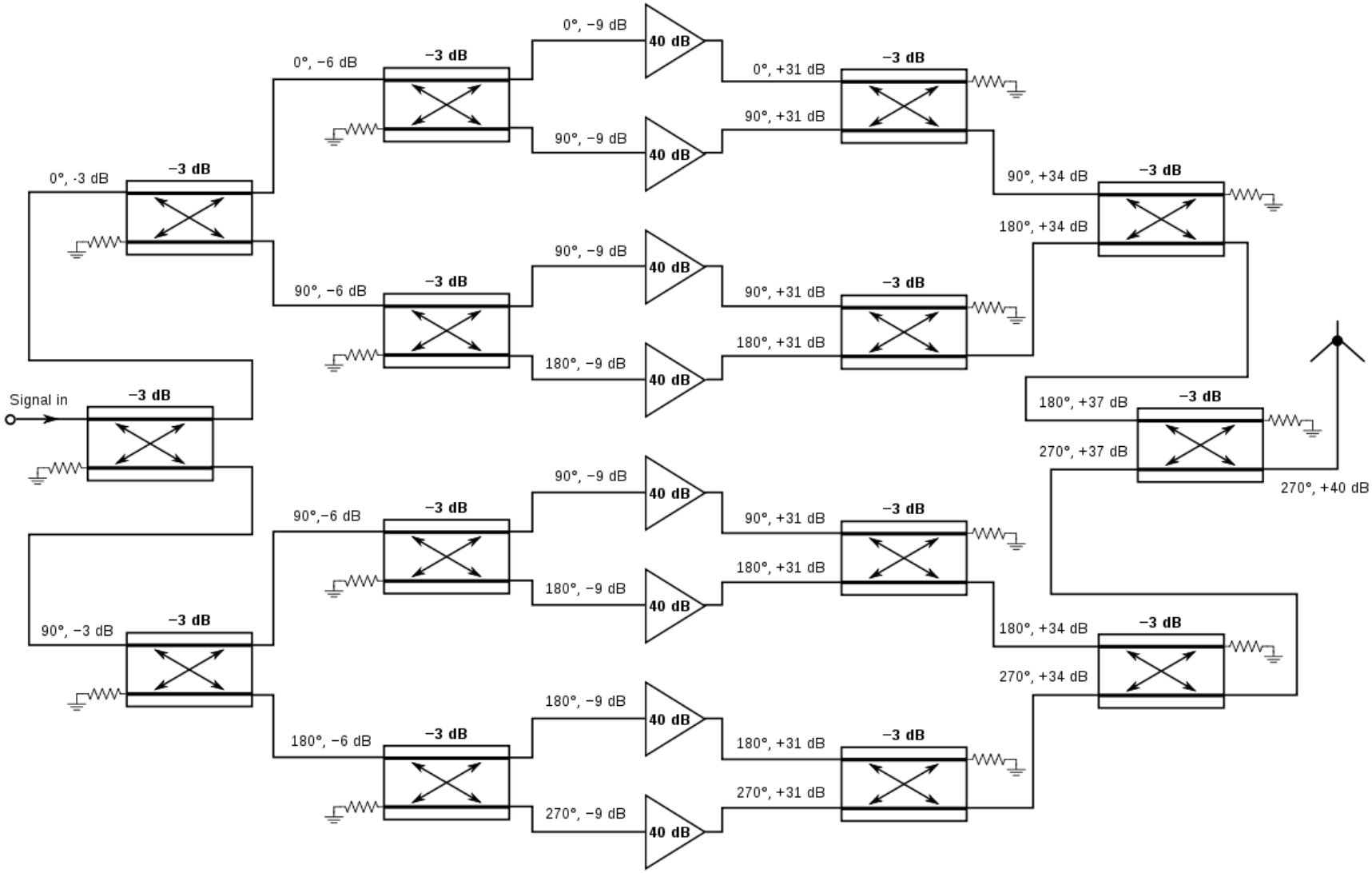
$$\mathrm{Im} z_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.335 \qquad \theta_{ss} = -\cot^{-1}(\mathrm{Im} z_S) = 36.8^\circ$$

Microwave Amplifiers

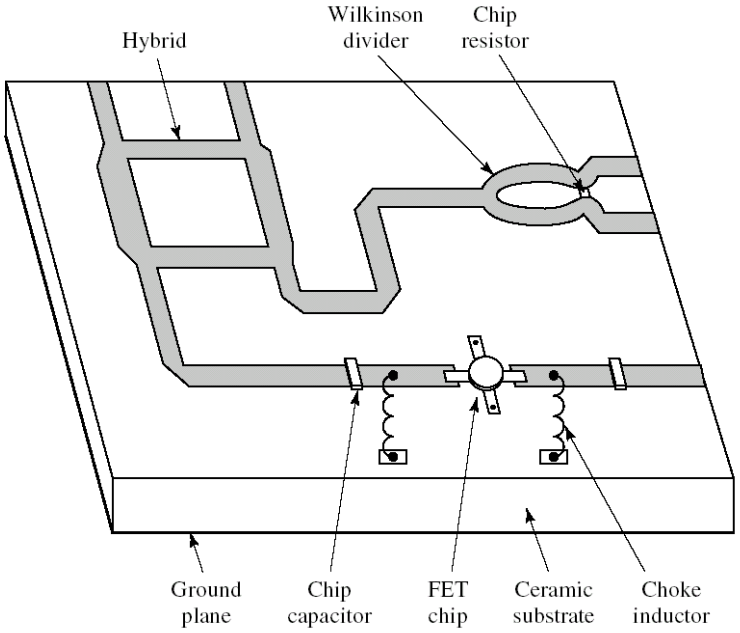
Microwave Amplifiers

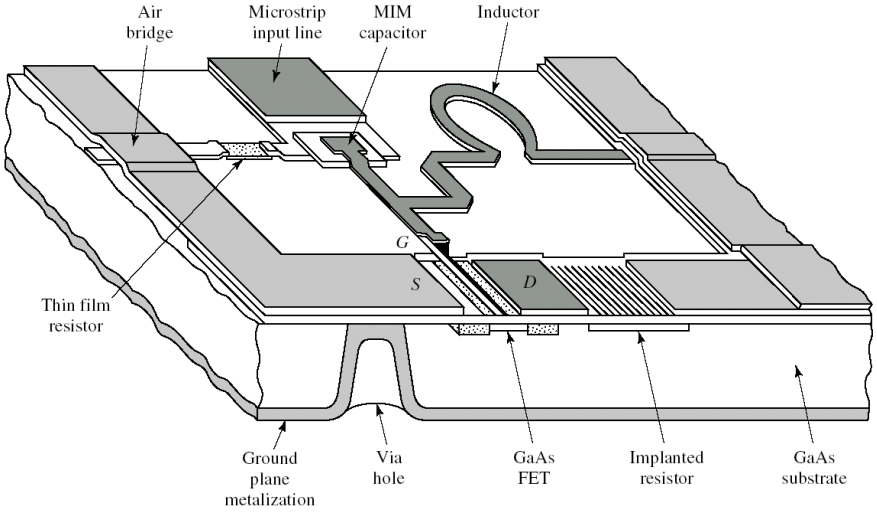


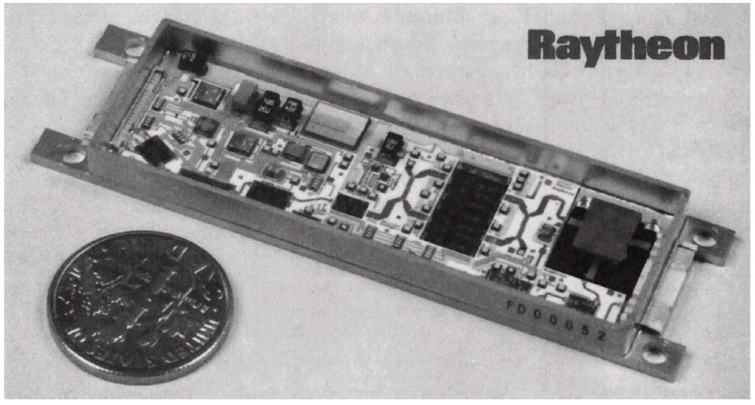
Balanced amplifiers

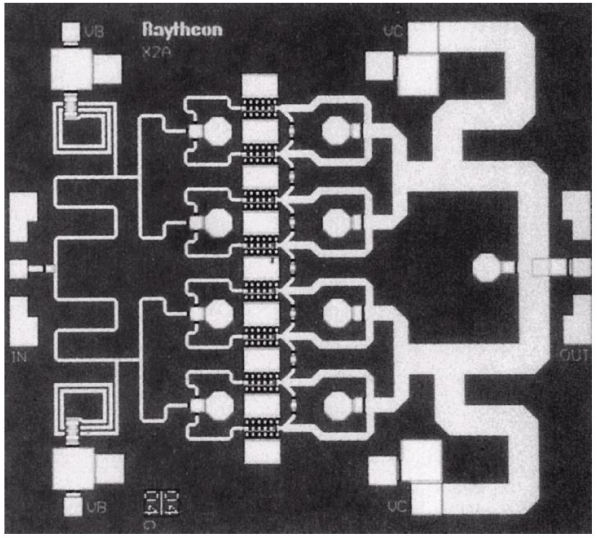


Microwave Integrated Circuits

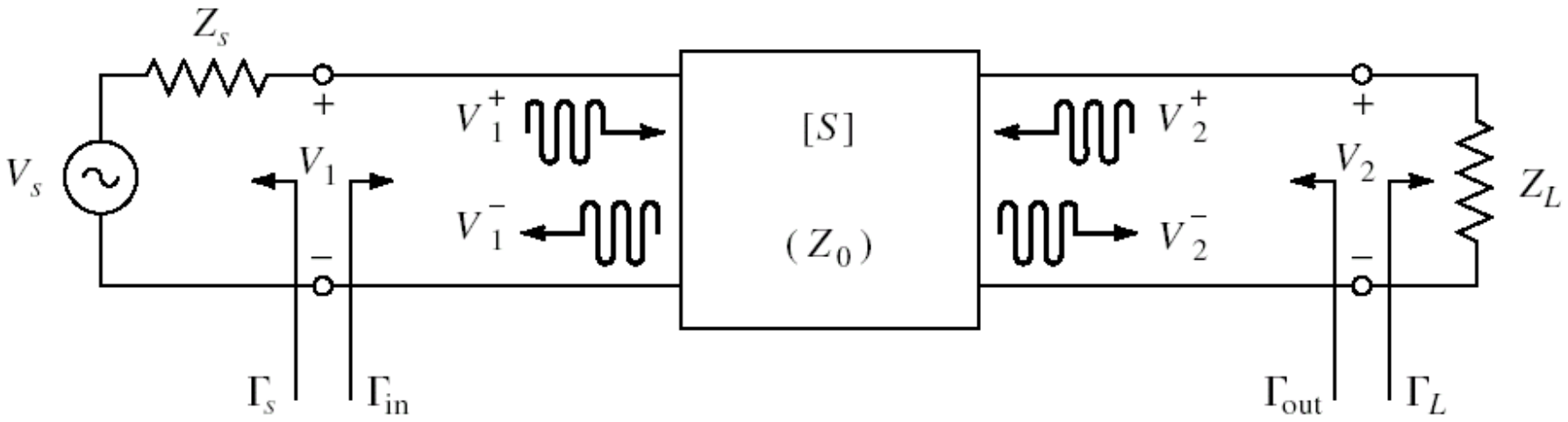








Amplifier as two-port



- Charaterized with S parameters
- normalized at Zo (implicit 50Ω)
- Datasheets: S parameters for specific bias conditions

Datasheets

CEL

NE46100 / NE46134

NPN MEDIUM POWER MICROWAVE TRANSISTOR

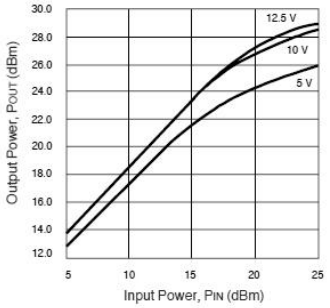
FEATURES

- **HIGH DYNAMIC RANGE**
- **LOW IM DISTORTION:** -40 dBc
- **HIGH OUTPUT POWER :** 27.5 dBm at TYP
- **LOW NOISE:** 1.5 dB TYP at 500 MHz
- **LOW COST**

DESCRIPTION

The NE461 series of NPN silicon epitaxial bipolar transistors is designed for medium power applications requiring high dynamic range. This device exhibits an outstanding combination of high gain and low intermodulation distortion, as well as low noise figure. The NE461 series offers excellent performance and reliability at low cost through titanium, platinum, gold metallization system and direct nitride passivation of the surface of the chip. Devices are available in a low cost surface mount package (SOT-89) as well as in chip form.

NE46134
TYPICAL OUTPUT POWER
vs. INPUT POWER
f = 1.0 GHz, Ic = 100 mA



ELECTRICAL CHARACTERISTICS (TA = 25°C)

PART NUMBER EIAJ[1] REGISTERED NUMBER PACKAGE OUTLINE			NE46100 00 (CHIP)			NE46134 2SC4536 34		
SYMBOLS	PARAMETERS AND CONDITIONS	UNITS	MIN	TYP	MAX	MIN	TYP	MAX
f_T	Gain Bandwidth Product at V_{CE} = 10 V, I_C = 100 mA	GHz		5.5			5.5	
NF_{MIN}	Minimum Noise Figure[3] at V_{CE} = 10 V, I_C = 50 mA, 500 MHz V_{CE} = 10 V, I_C = 50 mA, 1 GHz	dB dB		1.5 2.0			1.5 2.0	
G_L	Linear Gain, V_{CE} = 12.5 V, I_C = 100 mA, 2.0 GHz V_{CE} = 12.5 V, I_C = 100 mA, 1.0 GHz	dB dB		9.0			8.0	
$\|S_{21E}\|^2$	Insertion Power Gain at 10 V, 50 mA, f = 1.0 GHz	dB		10.0		5.5	7.0	
h_{FE}	DC Current Gain[2] at V_{CE} = 10 V, I_C = 50 mA		40		200	40		200
I_{CBO}	Collector Cutoff Current at V_{CB} = 20 V, I_E = 0 mA	μA			5.0			5.0
I_{EBO}	Emitter Cutoff Current at V_{EB} = 2 V, I_C = 0 mA	μA			5.0			5.0
P_{1dB}	Output Power at 1 dB Compression, V_{CE} = 12.5 V, I_C = 100 mA, 2.0 GHz V_{CE} = 12.5 V, I_C = 100 mA, 1.0 GHz	dBm dBm	27.0				27.5	
IM_3	Intermodulation Distortion, 10 V, 100 mA, F1 = 1.0 GHz, F2 = 0.99 GHz,							

Datasheets

NE46100

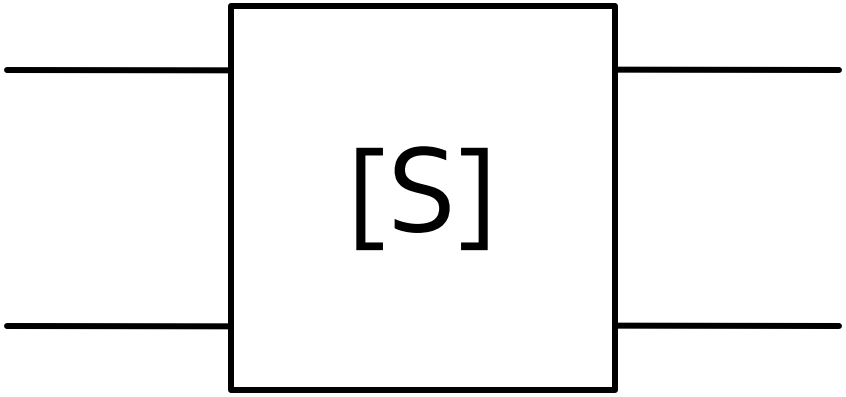
VCE = 5 V, IC = 50 mA

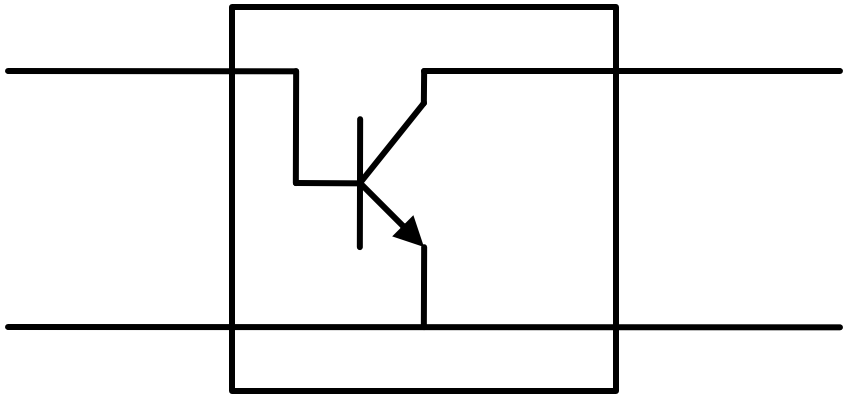
FREQUENCY	S_{11}		S_{21}		S_{12}		S_{22}		K	MAG^2
(MHz)	MAG	ANG	MAG	ANG	MAG	ANG	MAG	ANG		(dB)
100	0.778	-137	26.776	114	0.028	30	0.555	-102	0.16	29.8
200	0.815	-159	14.407	100	0.035	29	0.434	-135	0.36	26.2
500	0.826	-177	5.855	84	0.040	38	0.400	-162	0.75	21.7
800	0.827	176	3.682	76	0.052	43	0.402	-169	0.91	18.5
1000	0.826	173	2.963	71	0.058	47	0.405	-172	1.02	16.3
1200	0.825	170	2.441	66	0.064	47	0.412	-174	1.08	14.0
1400	0.820	167	2.111	61	0.069	47	0.413	-176	1.17	12.4
1600	0.828	165	1.863	57	0.078	54	0.426	-177	1.15	11.4
1800	0.827	162	1.671	53	0.087	50	0.432	-178	1.14	10.6
2000	0.828	159	1.484	49	0.093	50	0.431	-180	1.17	9.5
2500	0.822	153	1.218	39	0.11	48	0.462	177	1.18	7.8
3000	0.818	148	1.010	30	0.135	46	0.490	174	1.16	6.3
3500	0.824	142	0.876	21	0.147	44	0.507	170	1.16	5.3
4000	0.812	137	0.762	13	0.168	38	0.535	167	1.14	4.3

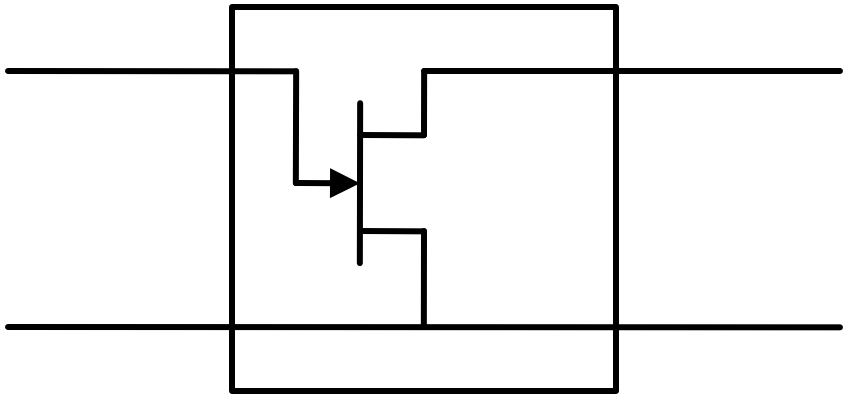
VCE = 5 V, IC = 100 mA

100	0.778	-144	27.669	111	0.027	35	0.523	-114	0.27	30.2
200	0.820	-164	14.559	97	0.029	29	0.445	-144	0.42	27.0
500	0.832	-179	5.885	84	0.035	38	0.435	-166	0.81	22.2
800	0.833	175	3.691	76	0.048	45	0.435	-173	0.95	18.8
1000	0.831	172	2.980	71	0.056	51	0.437	-176	1.05	16.0
1200	0.836	169	2.464	67	0.061	52	0.432	-178	1.11	14.0
1400	0.829	166	2.121	61	0.072	53	0.447	-180	1.12	12.6
1600	0.831	164	1.867	58	0.080	54	0.445	179	1.14	11.4

S parameters for transistors



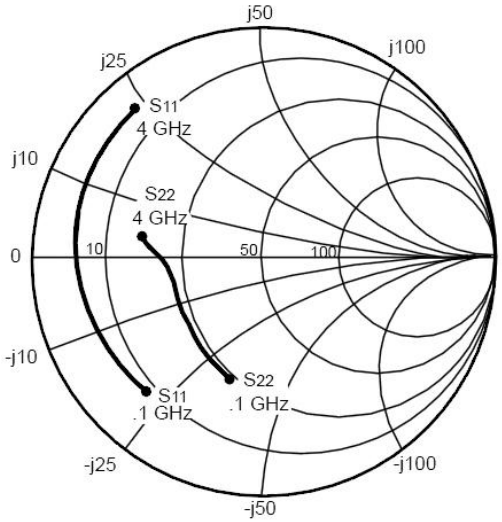


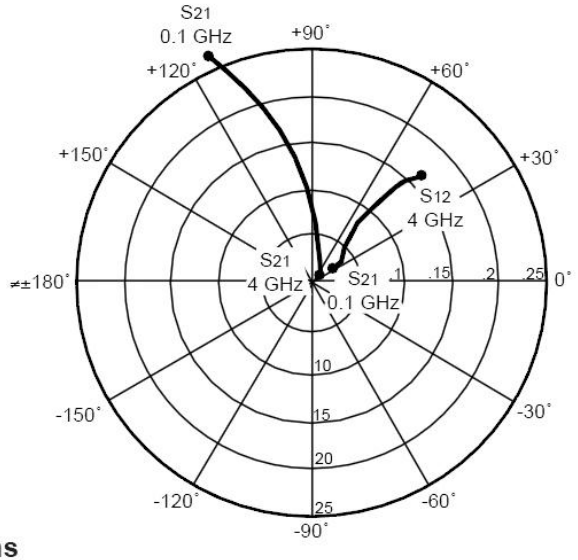


Datasheets

NE46100, NE46134

TYPICAL COMMON EMITTER SCATTERING PARAMETERS[1] (T_A = 25°C)





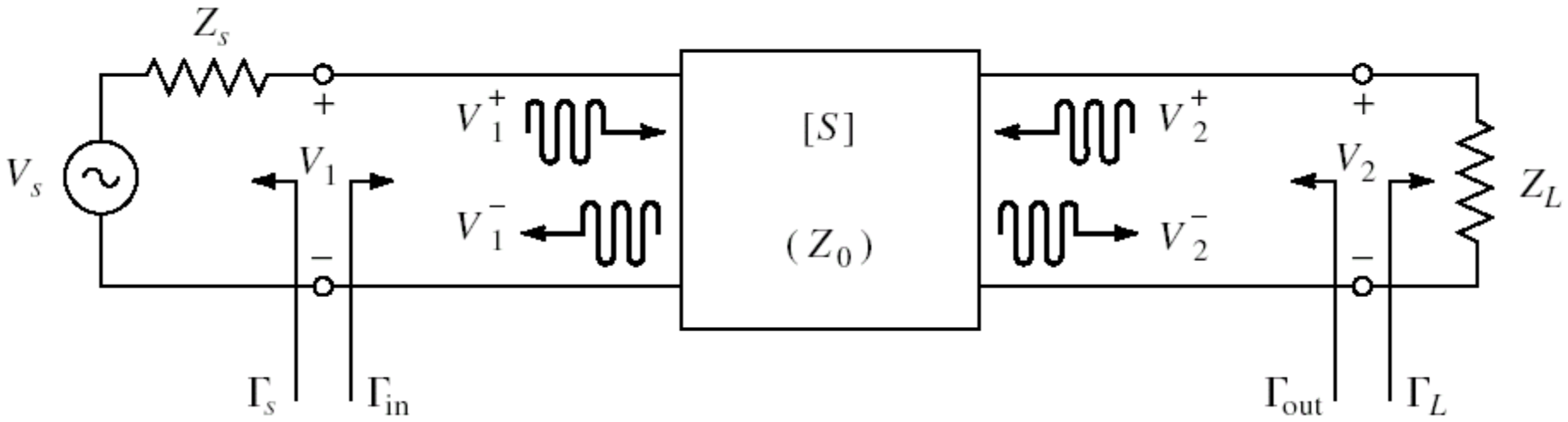
Coordinates in Ohms
Frequency in GHz
V_{CE} = 5 V, I_C = 50 mA

S2P - Touchstone

- Touchstone file format (*.s2p)

```
! SIEMENS Small Signal Semiconductors
! VDS = 3.5 V     ID = 15 mA
# GHz  S  MA  R  50
!  f        S11          S21          S12          S22
! GHz     MAG   ANG      MAG   ANG      MAG   ANG      MAG   ANG
 1.000 0.9800  -18.0   2.230  157.0  0.0240   74.0  0.6900  -15.0
 2.000 0.9500  -39.0   2.220  136.0  0.0450   57.0  0.6600  -30.0
 3.000 0.8900  -64.0   2.210  110.0  0.0680   40.0  0.6100  -45.0
 4.000 0.8200  -89.0   2.230   86.0  0.0850   23.0  0.5600  -62.0
 5.000 0.7400 -115.0   2.190   61.0  0.0990    7.0  0.4900  -80.0
 6.000 0.6500 -142.0   2.110   36.0  0.1070  -10.0  0.4100  -98.0
!
!  f      Fmin   Gammaopt  rn/50
! GHz       dB    MAG  ANG    -
 2.000     1.00  0.72   27   0.84
 4.000     1.40  0.64   61   0.58
```

Amplifier as two-port



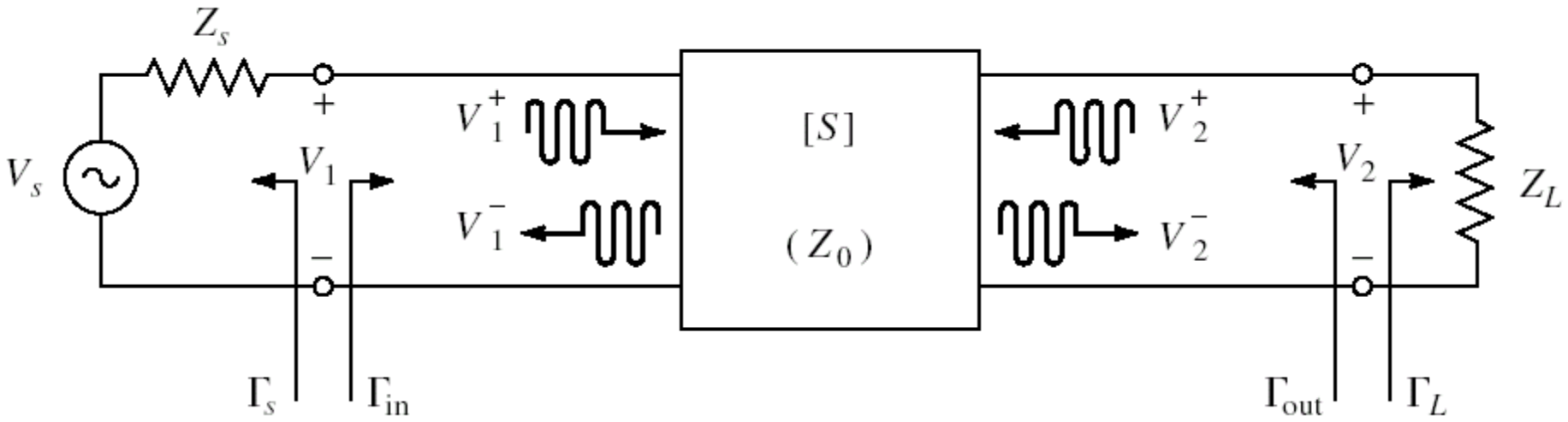
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} \qquad \begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$\Gamma_L = \frac{V_2^+}{V_2^-}$$

$$V_1^- = S_{11} \cdot V_1^+ + S_{12} \cdot V_2^+ = S_{11} \cdot V_1^+ + S_{12} \cdot \Gamma_L \cdot V_2^-$$

$$V_2^- = S_{21} \cdot V_1^+ + S_{22} \cdot V_2^+ = S_{21} \cdot V_1^+ + S_{22} \cdot \Gamma_L \cdot V_2^-$$

Amplifier as two-port



$$V_1^- = S_{11} \cdot V_1^+ + S_{12} \cdot V_2^+ = S_{11} \cdot V_1^+ + S_{12} \cdot \Gamma_L \cdot V_2^-$$

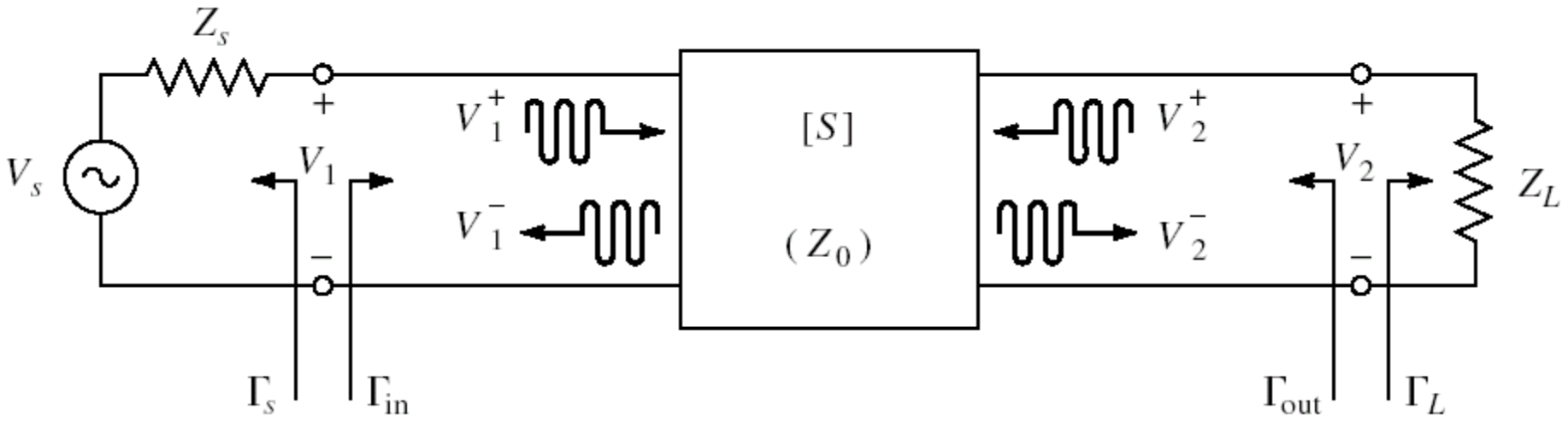
$$V_2^- = S_{21} \cdot V_1^+ + S_{22} \cdot V_2^+ = S_{21} \cdot V_1^+ + S_{22} \cdot \Gamma_L \cdot V_2^-$$

- similarly

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

Amplifier as two-port

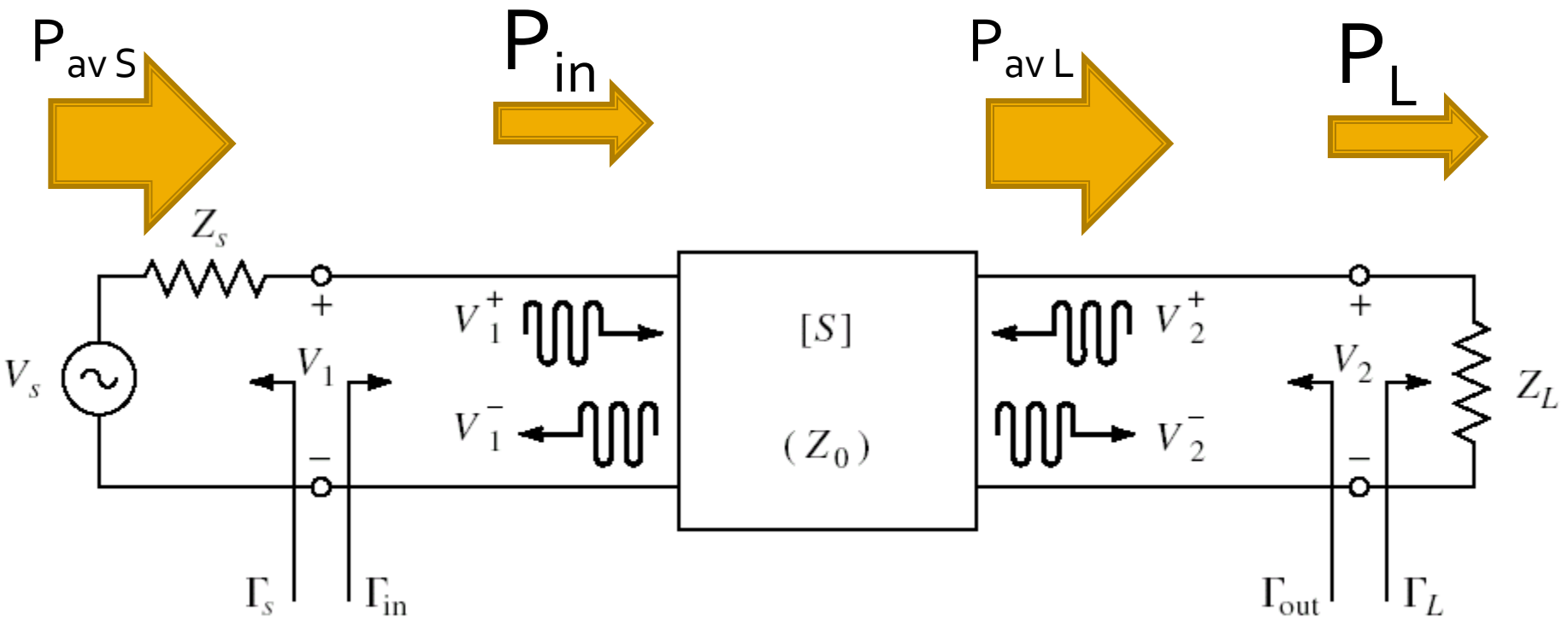


$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

Power / Matching

- Two ports in which matching influences the power transfer



Signal power

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$V_1 = \frac{V_S \cdot Z_{in}}{Z_S + Z_{in}} = V_1^+ + V_1^- = V_1^+ \cdot (1 + \Gamma_{in})$$

$$V_1^+ = \frac{V_S}{2} \frac{(1 - \Gamma_S)}{(1 - \Gamma_S \cdot \Gamma_{in})}$$

- L2

$$P_{in} = \frac{1}{2 \cdot Z_0} \cdot \left|V_1^+\right|^2 \cdot \left(1 - \left|\Gamma_{in}\right|^2\right)$$

$$P_L = \frac{1}{2 \cdot Z_0} \cdot \left|V_2^-\right|^2 \cdot \left(1 - \left|\Gamma_L\right|^2\right)$$

$$P_{in} = \frac{\left|V_S\right|^2}{8 \cdot Z_0} \cdot \frac{\left|1 - \Gamma_S\right|^2}{\left|1 - \Gamma_S \cdot \Gamma_{in}\right|^2} \left(1 - \left|\Gamma_{in}\right|^2\right)$$

$$V_2^- = S_{21} \cdot V_1^+ + S_{22} \cdot V_2^+ = S_{21} \cdot V_1^+ + S_{22} \cdot \Gamma_L \cdot V_2^-$$

$$V_2^- = \frac{S_{21} \cdot V_1^+}{1 - S_{22} \cdot \Gamma_L}$$

$$P_L = \frac{\left|V_1^+\right|^2}{2 \cdot Z_0} \cdot \frac{\left|S_{21}\right|^2}{\left|1 - S_{22} \cdot \Gamma_L\right|^2} \left(1 - \left|\Gamma_L\right|^2\right)$$

$$P_L = \frac{\left|V_S\right|^2}{8 \cdot Z_0} \cdot \frac{\left|S_{21}\right|^2 \cdot \left(1 - \left|\Gamma_L\right|^2\right)}{\left|1 - S_{22} \cdot \Gamma_L\right|^2} \cdot \frac{\left|1 - \Gamma_S\right|^2}{\left|1 - \Gamma_S \cdot \Gamma_{in}\right|^2}$$

Signal power

- Signal power

$$P_{in} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|1-\Gamma_S|^2}{|1-\Gamma_S \cdot \Gamma_{in}|^2} \left(1-|\Gamma_{in}|^2\right)$$

$$P_L = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_L|^2\right)}{|1-S_{22} \cdot \Gamma_L|^2} \cdot \frac{|1-\Gamma_S|^2}{|1-\Gamma_S \cdot \Gamma_{in}|^2}$$

- Power available from the source

$$P_{av\,S} = P_{in}\big|_{\Gamma_{in}=\Gamma_S^*} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|1-\Gamma_S|^2}{\left(1-|\Gamma_S|^2\right)}$$

- Power available on the load (from the network)

$$P_{av\,L} = P_L\big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_S|^2}{8 \cdot Z_0} \cdot \frac{|S_{21}|^2 \cdot |1-\Gamma_S|^2}{|1-S_{11} \cdot \Gamma_S|^2 \cdot \left(1-|\Gamma_{out}|^2\right)}$$

Two-Port Power Gains

- Power Gain

$$G = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2 \cdot \left(1 - |\Gamma_L|^2\right)}{\left(1 - |\Gamma_{in}|^2\right) \cdot |1 - S_{22} \cdot \Gamma_L|^2}$$

$$P_{in} = P_{in}\left(\Gamma_S, \Gamma_{in}\left(\Gamma_L\right), S\right)$$

$$P_L = P_L\left(\Gamma_S, \Gamma_{in}\left(\Gamma_L\right), S\right)$$

- The **actual** power gain **introduced** by the amplifier is less important because a higher gain may be accompanied by a **decrease** in input power (power actually drained from the source)
- We prefer to characterize the amplifier effect looking to the **power actually delivered to the load** in relation to the power **available from the source** (which is a constant)

Two-Port Power Gains

- **Available** power gain

$$G_A = \frac{P_{av\,L}}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)}{|1-S_{22}\cdot\Gamma_L|^2 \cdot \left(1-|\Gamma_{out}|^2\right)}$$

- **Transducer** power gain

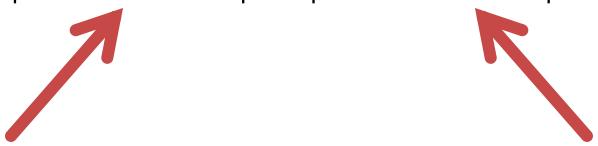
$$G_T = \frac{P_L}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)\cdot\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\cdot\Gamma_{in}|^2 \cdot |1-S_{22}\cdot\Gamma_L|^2}$$

$$\Gamma_{in} = \Gamma_{in}(\Gamma_L)$$

- **Unilateral transducer** power gain

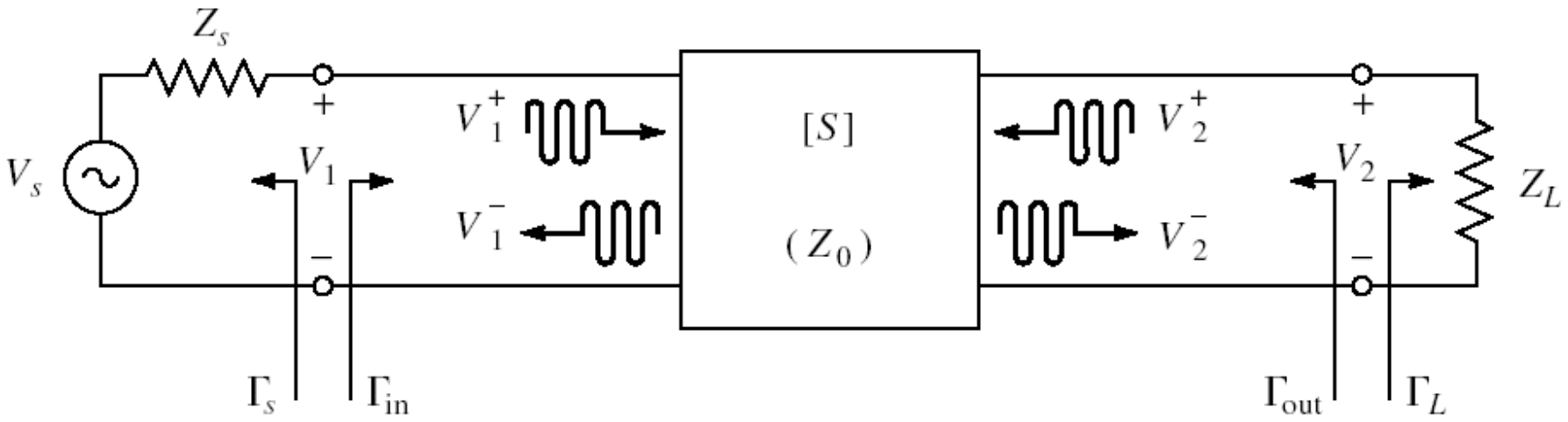
$$G_{TU} = |S_{21}|^2 \cdot \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$S_{12} \cong 0 \qquad \Gamma_{in} = S_{11}$$



Input and output can be treated independently

Amplifier as two-port

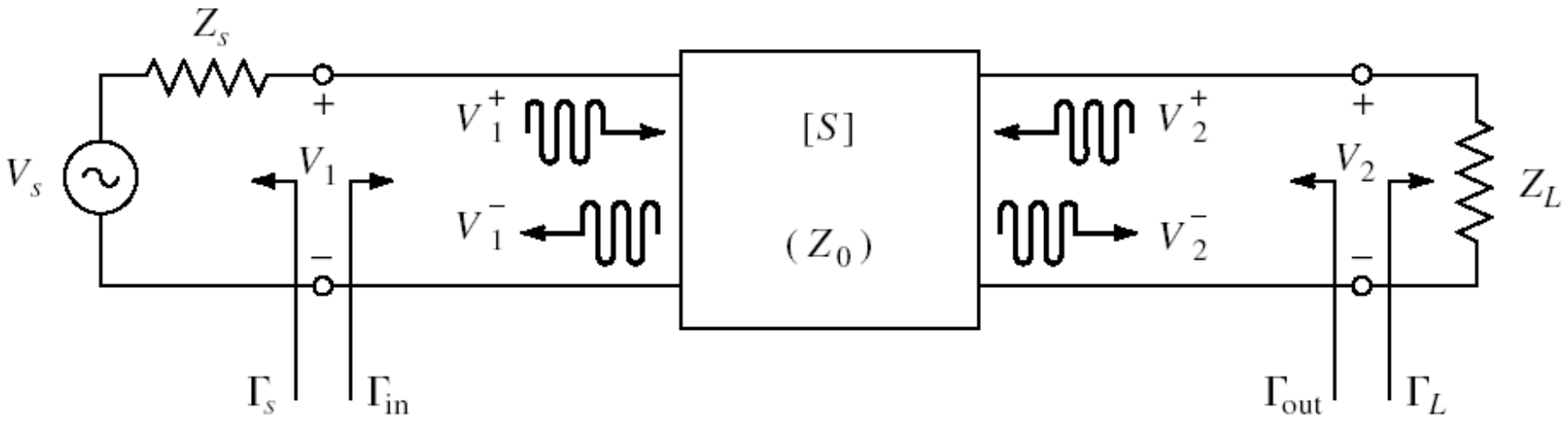


- For an amplifier two-port we are interested in:
  - stability
  - power gain
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Microwave Amplifiers

Stability

Amplifier as two-port



- For an amplifier two-port we are interested in:
  - **stability**
  - power gain
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Stability

- L7 $\Gamma = \Gamma_r + j \cdot \Gamma_i$ $r_L = \dfrac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}$

  Z_{in} $\Gamma_{in} = \Gamma_r + j \cdot \Gamma_i$

- instability

  $\mathrm{Re}\{Z_{in}\} < 0 \Leftrightarrow 1-\Gamma_r^2-\Gamma_i^2 < 0$ $\Gamma_r^2+\Gamma_i^2 > 1$ $|\Gamma_{in}| > 1$

- stability, Z_{in}
  - conditions to be met by Γ_L to achieve (input) stability

    $|\Gamma_{in}| < 1$ $\left| S_{11} + \dfrac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$
- similarly Z_{out}
  - conditions to be met by Γ_S to achieve (output) stability

Stability

$$|\Gamma_{in}| < 1 \qquad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$$

- We can calculate conditions to be met by Γ_L to achieve stability

$$|\Gamma_{out}| < 1 \qquad \left| S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S} \right| < 1$$

- We can calculate conditions to be met by Γ_S to achieve stability

Stability

$$|\Gamma_{in}| < 1 \qquad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| < 1$$

- The limit between stability/instability

$$|\Gamma_{in}| = 1 \qquad \left| S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \right| = 1$$

$$\left| S_{11} \cdot (1 - S_{22} \cdot \Gamma_L) + S_{12} \cdot S_{21} \cdot \Gamma_L \right| = \left| 1 - S_{22} \cdot \Gamma_L \right|$$

- determinant of the S matrix $\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21}$

$$\left| S_{11} - \Delta \cdot \Gamma_L \right| = \left| 1 - S_{22} \cdot \Gamma_L \right|$$

$$\left| S_{11} - \Delta \cdot \Gamma_L \right|^2 = \left| 1 - S_{22} \cdot \Gamma_L \right|^2$$

Stability

$$\left|S_{11}-\Delta\cdot\Gamma_L\right|^2=\left|1-S_{22}\cdot\Gamma_L\right|^2$$

$$a\cdot a^*=\left|a\right|\cdot e^{j\theta}\cdot\left|a\right|\cdot e^{-j\theta}=\left|a\right|^2$$

$$\left|a+b\right|^2=(a+b)\cdot(a+b)^*=(a+b)\cdot\left(a^*+b^*\right)=\left|a\right|^2+\left|b\right|^2+a^*\cdot b+a\cdot b^*$$

$$\left|S_{11}\right|^2+\left|\Delta\right|^2\cdot\left|\Gamma_L\right|^2-\left(\Delta\cdot\Gamma_L\cdot S_{11}^*+\Delta^*\cdot\Gamma_L^*\cdot S_{11}\right)=1+\left|S_{22}\right|^2\cdot\left|\Gamma_L\right|^2-\left(S_{22}^*\cdot\Gamma_L^*+S_{22}\cdot\Gamma_L\right)$$

$$\left(\left|S_{22}\right|^2-\left|\Delta\right|^2\right)\cdot\Gamma_L\cdot\Gamma_L^*-\left(S_{22}-\Delta\cdot S_{11}^*\right)\cdot\Gamma_L-\left(S_{22}^*-\Delta^*\cdot S_{11}\right)\cdot\Gamma_L^*=\left|S_{11}\right|^2-1$$

$$\Gamma_L\cdot\Gamma_L^*-\frac{\left(S_{22}-\Delta\cdot S_{11}^*\right)\cdot\Gamma_L+\left(S_{22}^*-\Delta^*\cdot S_{11}\right)\cdot\Gamma_L^*}{\left|S_{22}\right|^2-\left|\Delta\right|^2}=\frac{\left|S_{11}\right|^2-1}{\left|S_{22}\right|^2-\left|\Delta\right|^2}$$

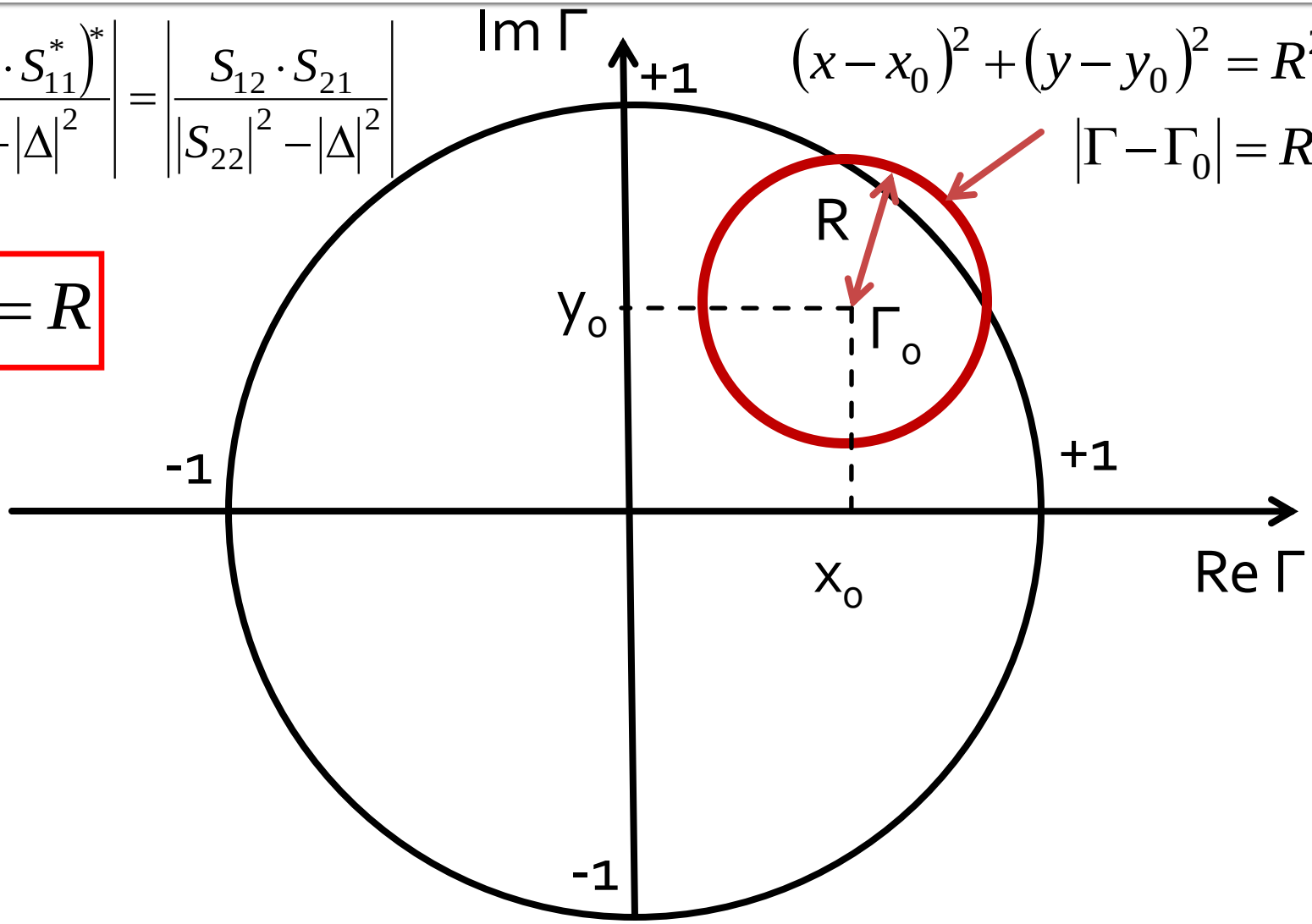
$$+\frac{\left|S_{22}-\Delta\cdot S_{11}^*\right|^2}{\left(\left|S_{22}\right|^2-\left|\Delta\right|^2\right)^2}$$

$$\left|\Gamma_L-\frac{\left(S_{22}-\Delta\cdot S_{11}^*\right)^*}{\left|S_{22}\right|^2-\left|\Delta\right|^2}\right|^2=\frac{\left|S_{11}\right|^2-1}{\left|S_{22}\right|^2-\left|\Delta\right|^2}+\frac{\left|S_{22}-\Delta\cdot S_{11}^*\right|^2}{\left(\left|S_{22}\right|^2-\left|\Delta\right|^2\right)^2}$$

Stability

$$\left|\Gamma_L - \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2}\right| = \left|\frac{S_{12} \cdot S_{21}}{|S_{22}|^2 - |\Delta|^2}\right|$$

$$|\Gamma - \Gamma_0| = R$$



Output stability circle (CSOUT)

$$\left|\Gamma_L - \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2}\right| = \left|\frac{S_{12} \cdot S_{21}}{|S_{22}|^2 - |\Delta|^2}\right| \qquad |\Gamma_L - C_L| = R_L$$

- We obtain the equation of a circle in the complex plane, which represents the locus of Γ_L for the **limit between stability and instability** ($|\Gamma_{in}| = 1$)
- This circle is the **output stability circle** (Γ_L)

$$C_L = \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2} \qquad R_L = \frac{|S_{12} \cdot S_{21}|}{\left||S_{22}|^2 - |\Delta|^2\right|}$$

Input stability circle (CSIN)

- Similarly

$$|\Gamma_{out}| = 1 \qquad \left| S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S} \right| = 1$$

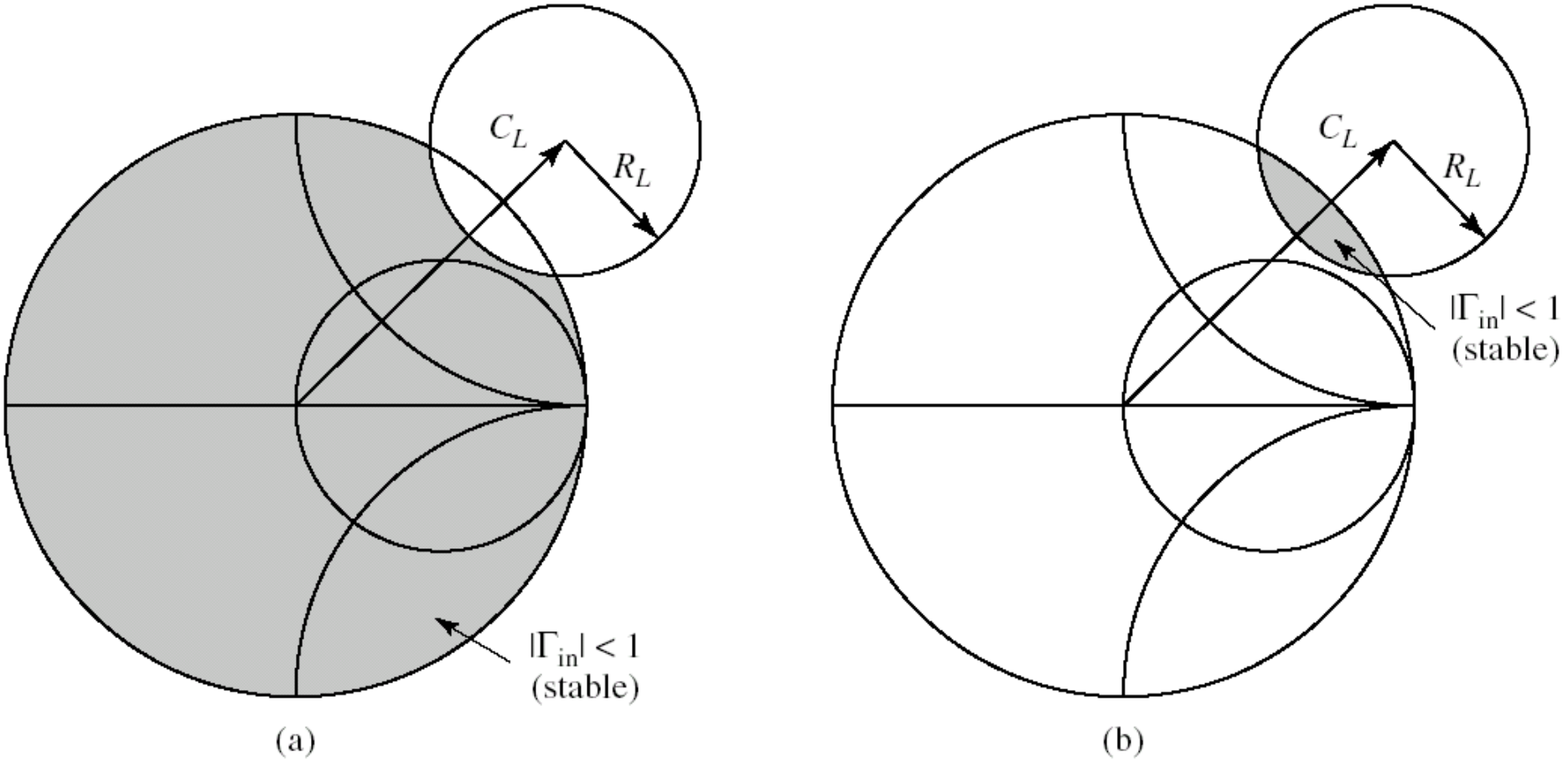
- We obtain the equation of a circle in the complex plane, which represents the locus of Γ_S for the **limit between stability and instability** ($|\Gamma_{out}| = 1$)
- This circle is the **input stability circle** (Γ_S)

$$C_S = \frac{\left(S_{11} - \Delta \cdot S_{22}^*\right)^*}{|S_{11}|^2 - |\Delta|^2} \qquad R_S = \frac{|S_{12} \cdot S_{21}|}{\left||S_{11}|^2 - |\Delta|^2\right|}$$

Output stability circle (CSOUT)

- The **output stability circle** represents the locus of Γ_L for the **limit between stability and instability** ($|\Gamma_{in}| = 1$)
- The circle divides the complex planes in two areas, the **inside** and the **outside** of the circle
- The two areas will represent the locus of Γ_L for stability ($|\Gamma_{in}|<1$) / instability ($|\Gamma_{in}|>1$)

Output stability circle (CSOUT)



- Two cases possible: (a) stable outside/ (b) stable inside

Output stability circle (CSOUT)

- Identification of the stability / instability regions
  - The center of the Smith Chart in Γ_L complex plane corresponds to $\Gamma_L = 0$
  - Input reflection coefficient

$$\Gamma_{in} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L} \qquad \Gamma_{in}\big|_{\Gamma_L = 0} = S_{11} \qquad \left|\Gamma_{in}\right|_{\Gamma_L = 0} = \left|S_{11}\right|$$

  - A decision can be made based on |S11| value and on the position of the center of the Smith chart (origin of the complex plane) relative to the circle

Identification of the stability / instability regions

- Output stability circle
  - **|S11| < 1** → the center of the Smith chart on which Γ_L is represented is a **stable point**, so it's placed in the stability region (most often situation)
  - |S11| > 1 → the center of the Smith chart on which Γ_L is represented is a **unstable point**, so it's placed in the instability region
- Input stability circle
  - **|S22| < 1** → the center of the Smith chart on which Γ_S is represented is a **stable point**, so it's placed in the stability region (most often situation)
  - |S22| > 1 → the center of the Smith chart on which Γ_S is represented is a **unstable point**, so it's placed in the instability region

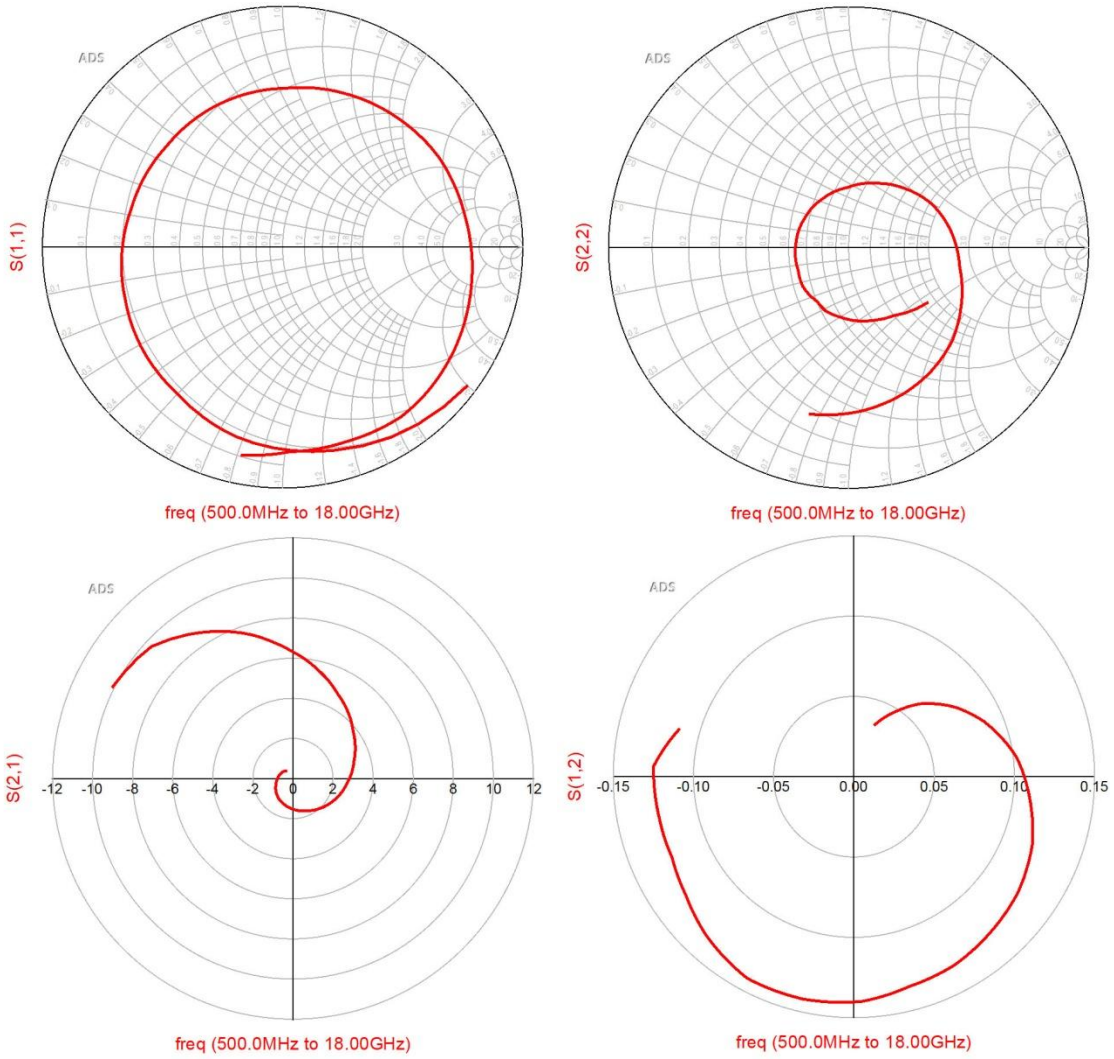
Example

- ATF-34143 **at Vds=3V Id=20mA.**
- @5GHz
  - S11 = 0.64∠139°
  - S12 = 0.119∠-21°
  - S21 = 3.165 ∠16°
  - S22 = 0.22 ∠146°

```
!ATF-34143
!S-PARAMETERS at Vds=3V  Id=20mA.   LAST UPDATED 01-29-99

# ghz s ma r 50

2.0  0.75  -126  6.306  90  0.088  23  0.26  -120
2.5  0.72  -145  5.438  75  0.095  15  0.25  -140
3.0  0.69  -162  4.762  62  0.102  7  0.23  -156
4.0  0.65  166  3.806  38  0.111  -8  0.22  174
5.0  0.64  139  3.165  16  0.119  -21  0.22  146
6.0  0.65  114  2.706  -5  0.125  -35  0.23  118
7.0  0.66  89  2.326  -27  0.129  -49  0.25  91
8.0  0.69  67  2.017  -47  0.133  -62  0.29  67
9.0  0.72  48  1.758  -66  0.135  -75  0.34  46

!FREQ   Fopt    GAMMA OPT      RN/Zo
!GHZ    dB     MAG     ANG     -

2.0  0.19  0.71  66  0.09
2.5  0.23  0.65  83  0.07
3.0  0.29  0.59  102  0.06
4.0  0.42  0.51  138  0.03
5.0  0.54  0.45  174  0.03
6.0  0.67  0.42  -151  0.05
7.0  0.79  0.42  -118  0.10
8.0  0.92  0.45  -88  0.18
9.0  1.04  0.51  -63  0.30
10.0  1.16  0.61  -43  0.46
```

Example

- ATF-34143
- **at**
  - **Vds=3V**
  - **Id=20mA**.



Solution + region identification

- S parameters
  - S11 = -0.483+0.42·j
  - S12 = 0.111-0.043·j
  - S21 = 3.042+0.872·j
  - S22 = -0.182+0.123·j
- $|S22| = 0.22 < 1$
- $|C_L| < R_L$, $o \in CSOUT$
- The center of the Smith chart is placed inside the output stability circle ($o \in CSOUT$) and is a stable point ($|S22| < 1$)
  - the inside of the output stability circle – stability region
  - the outside of the output stability circle – instability region

$$C_L = \frac{\left(S_{22} - \Delta \cdot S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2} = 3.931 - 0.897 \cdot j$$

$$|C_L| = 4.032$$

$$R_L = \frac{|S_{12} \cdot S_{21}|}{\left||S_{22}|^2 - |\Delta|^2\right|} = 4.891$$

Solution + region identification

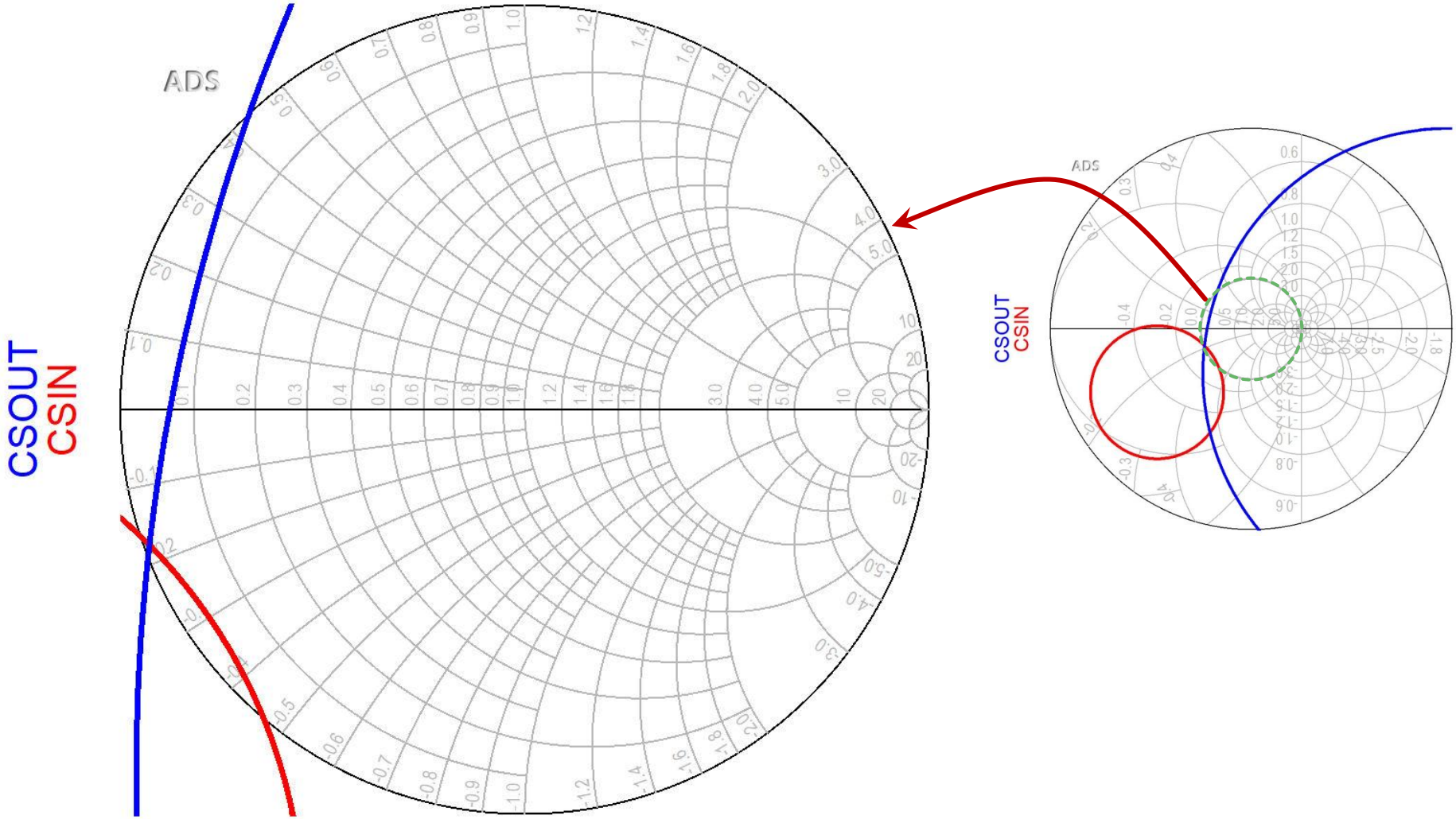
- S parameters
  - S11 = -0.483+0.42·j
  - S12 = 0.111-0.043·j
  - S21 = 3.042+0.872·j
  - S22 = -0.182+0.123·j
- $|S11| = 0.64 < 1$
- $|C_S| > R_S$, $0 \notin CSIN$
- The center of the Smith chart is placed outside the input stability circle ($0 \notin CSIN$) and is a stable point ($|S11| < 1$)
  - the outside of the input stability circle – stability region
  - the inside of the input stability circle – instability region

$$C_S = \frac{\left(S_{11} - \Delta \cdot S_{22}^*\right)^*}{|S_{11}|^2 - |\Delta|^2} = -1.871 - 1.265 \cdot j$$

$$|C_S| = 2.259$$

$$R_S = \frac{|S_{12} \cdot S_{21}|}{\left||S_{11}|^2 - |\Delta|^2\right|} = 1.325$$

ADS

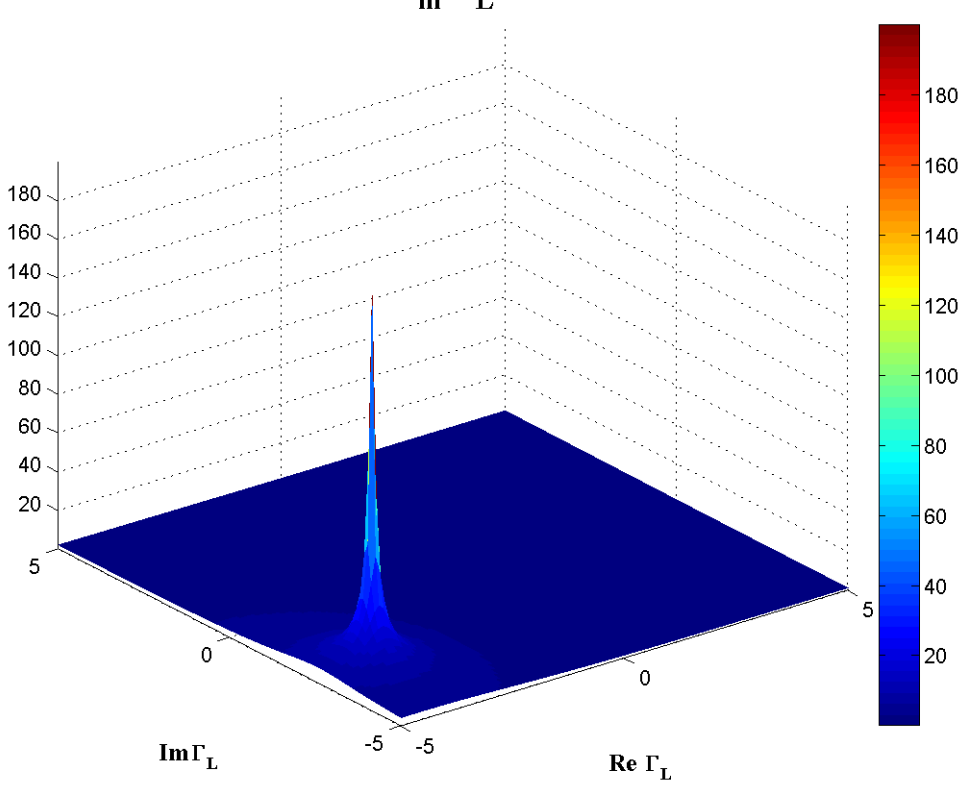


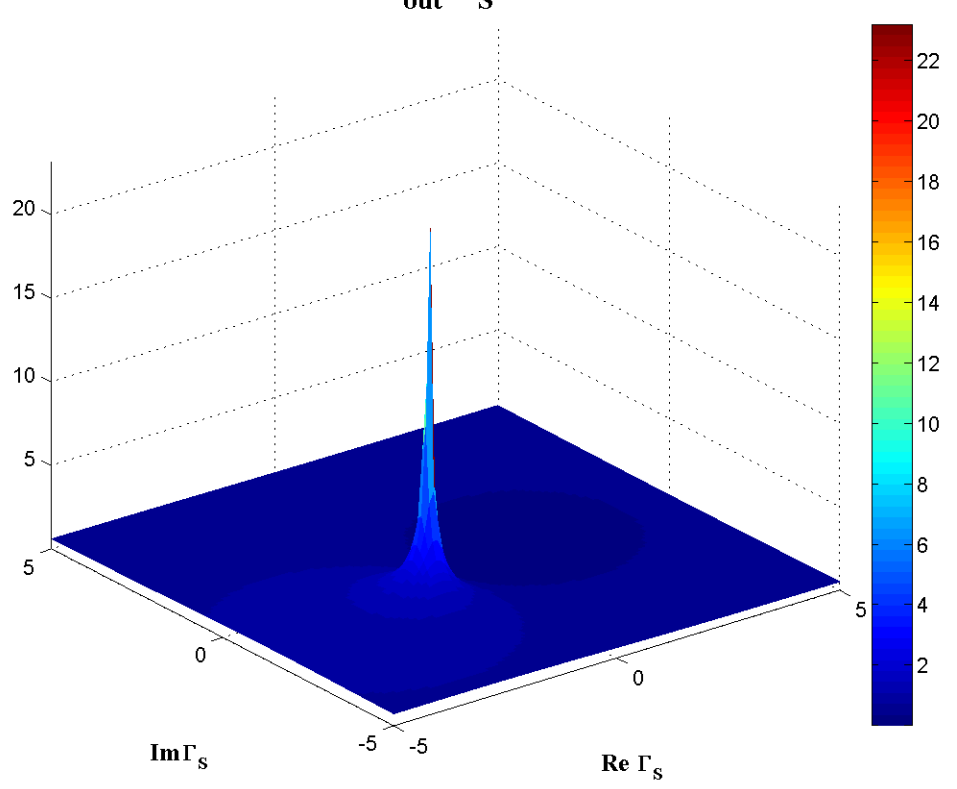
3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$

- High variations -> we change to z logarithmic scale

$$\Gamma_{in}(\Gamma_L) = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

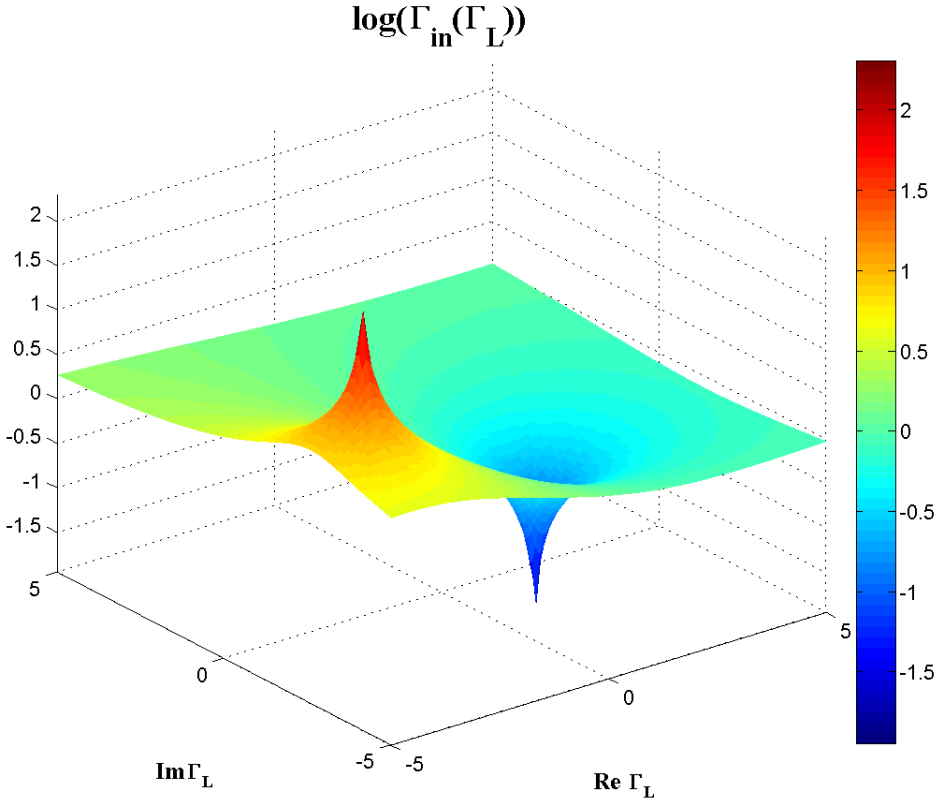
$$\Gamma_{out}(\Gamma_S) = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

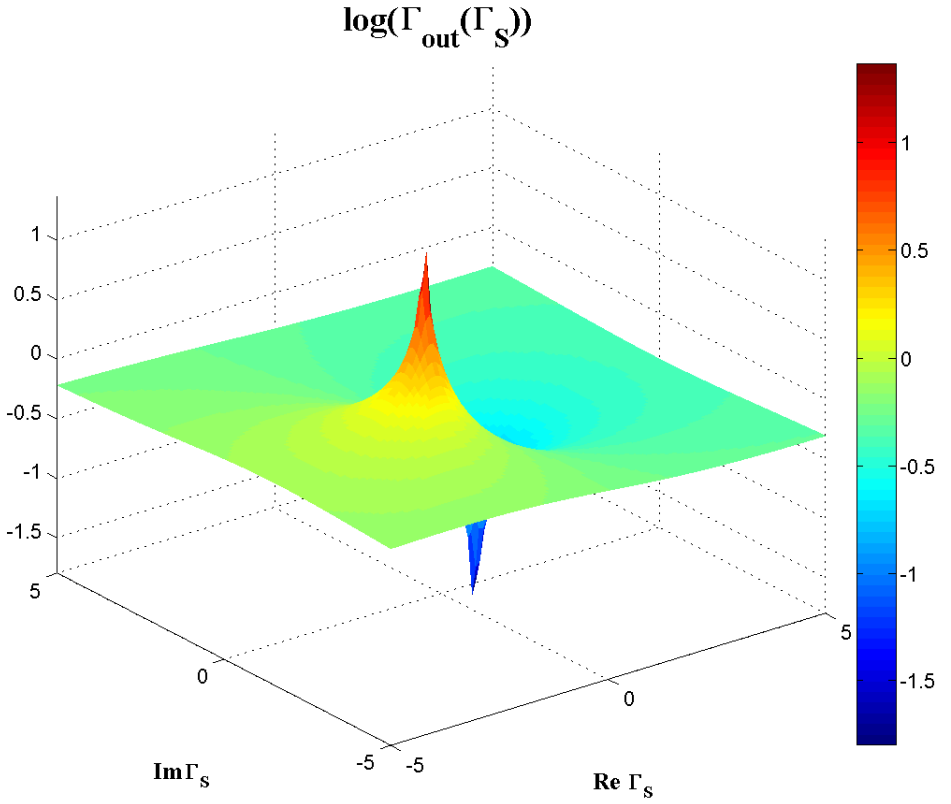




3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$

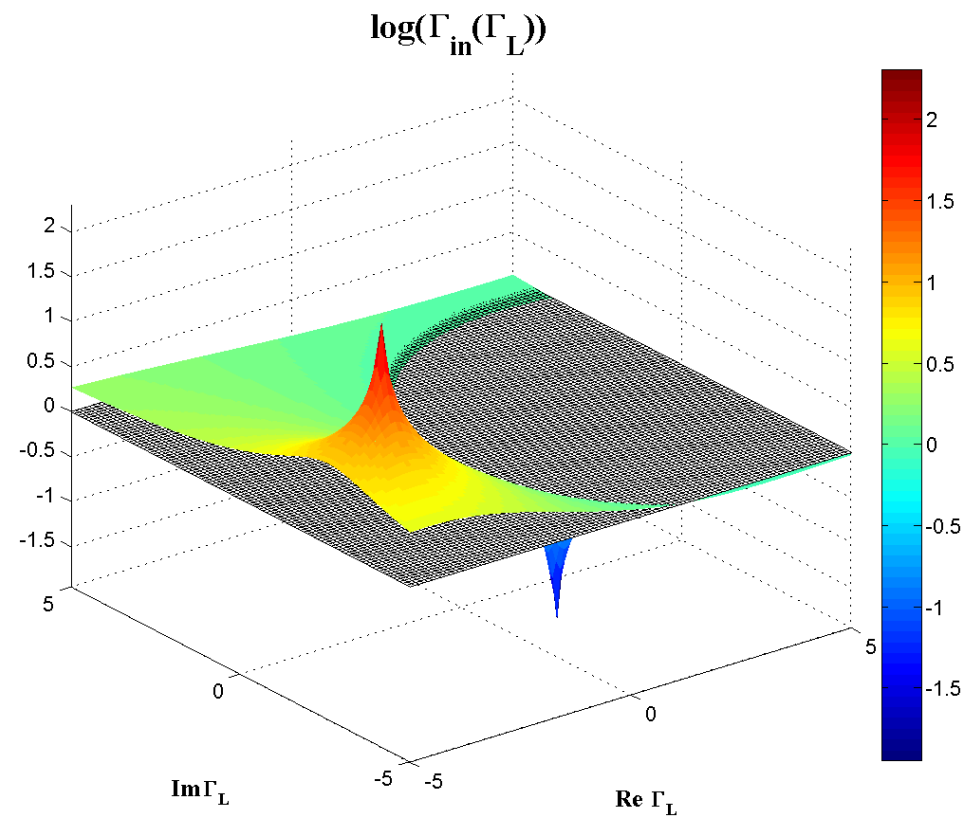
- $\log_{10}|\Gamma_{in}|$, $\log_{10}|\Gamma_{out}|$

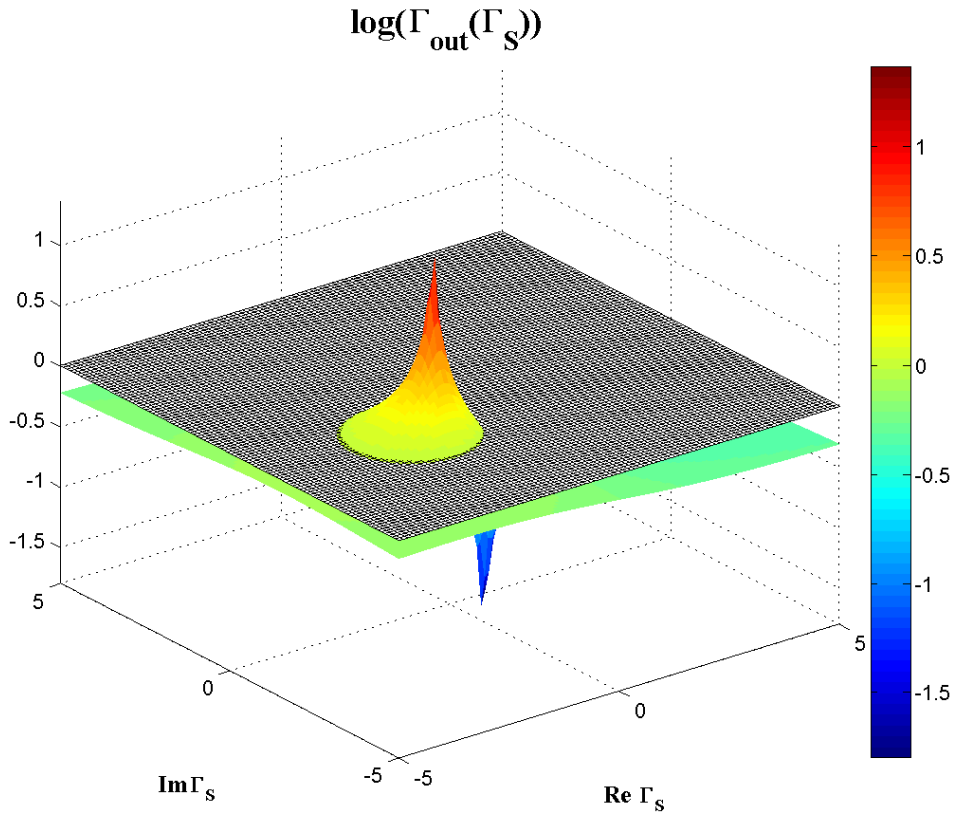




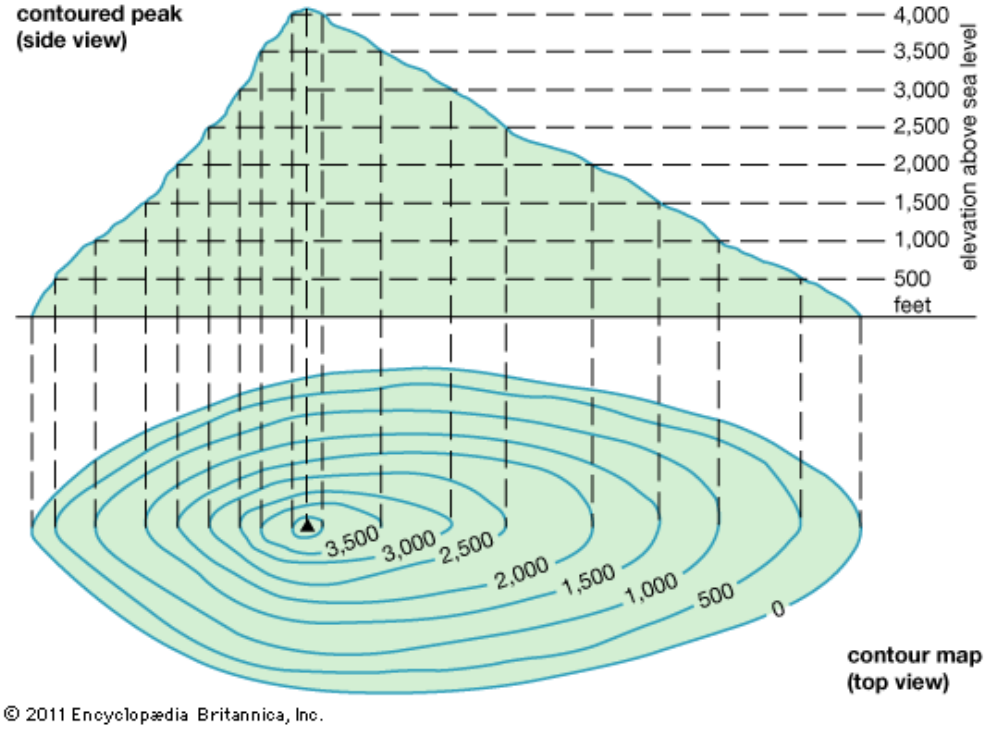
3D representation of $|\Gamma_{in}|$, $|\Gamma_{out}|$, $|\Gamma|=1$

- $|\Gamma| = 1 \rightarrow \log_{10}|\Gamma| = 0$, the intersection with the plane z = 0 is a circle





Contour map/lines



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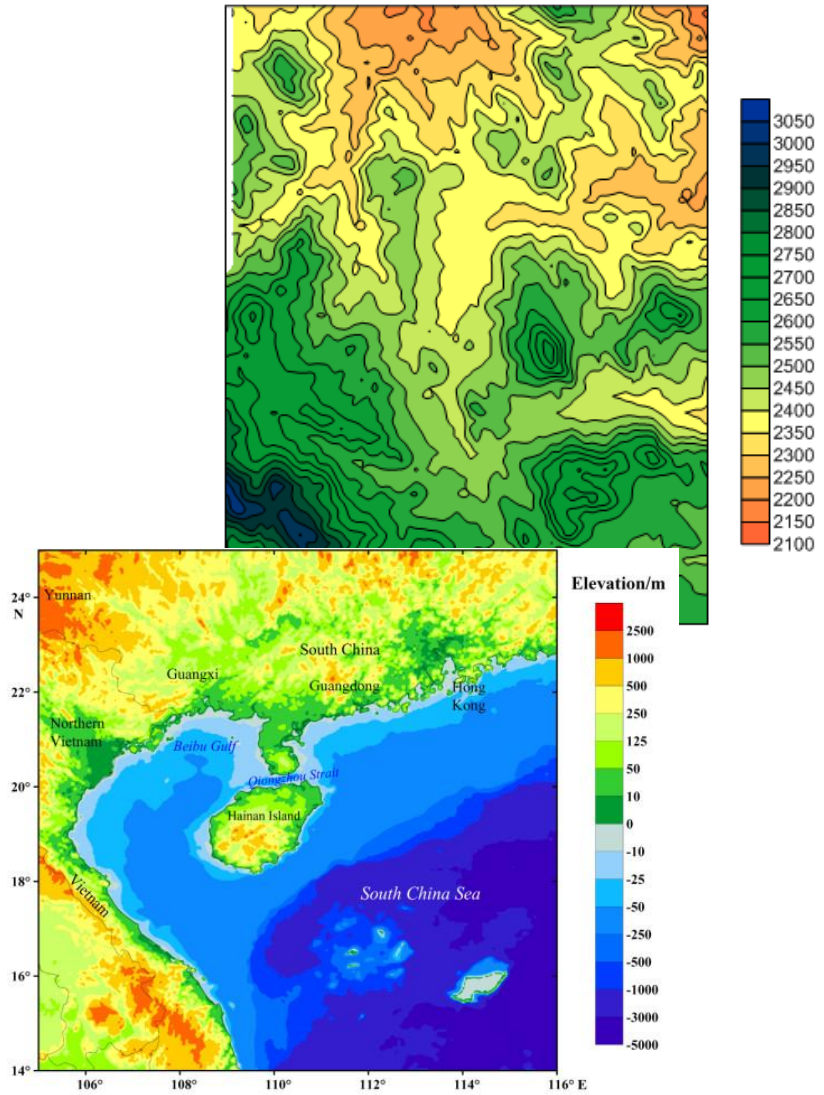
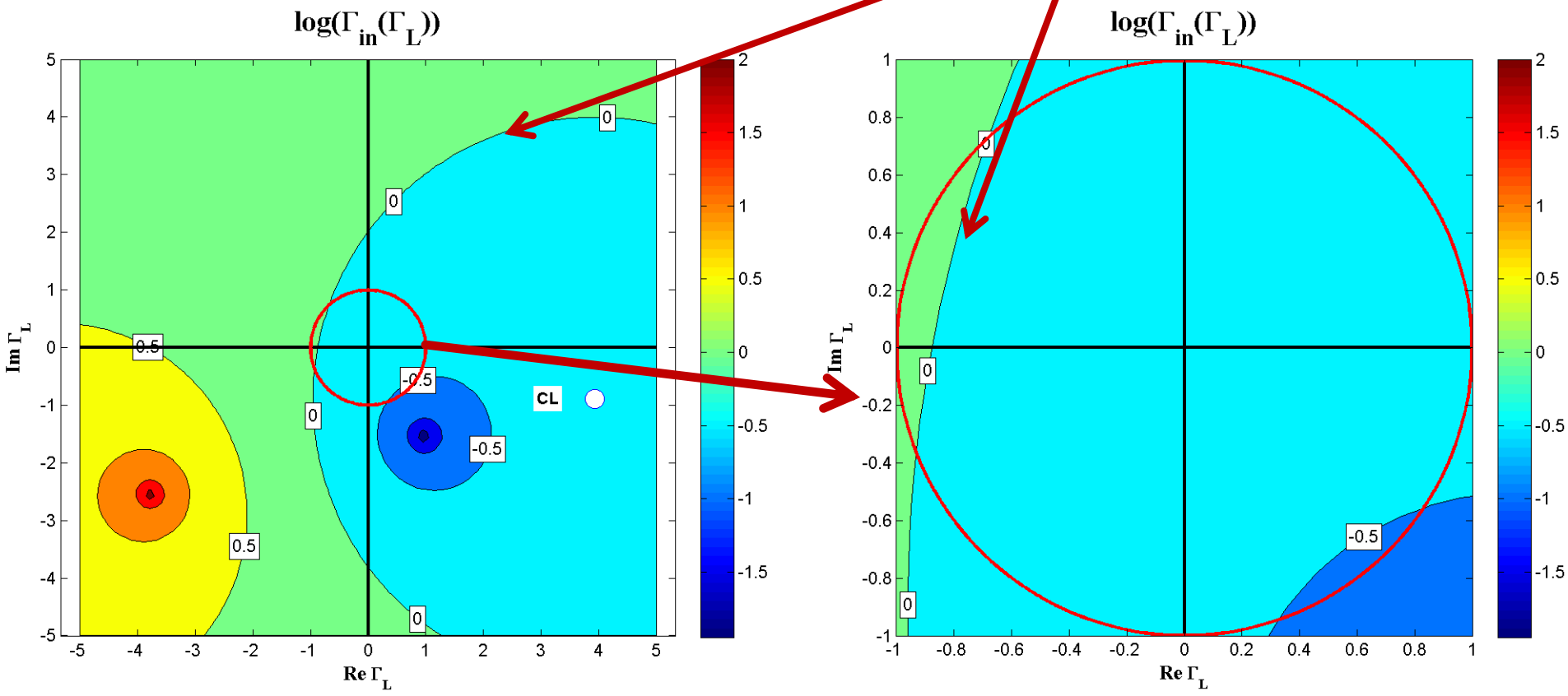


Contour lines of $\log_{10}|\Gamma_{in}|$

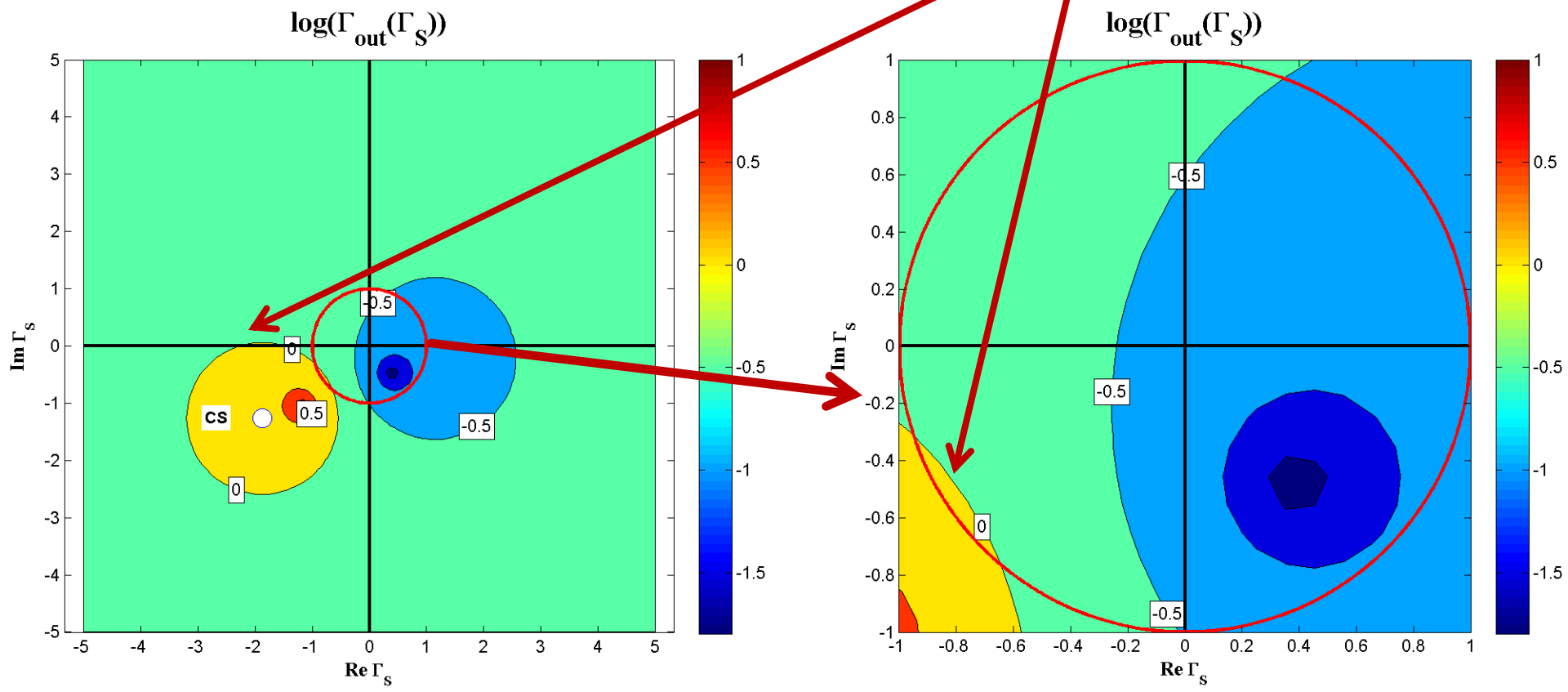
- $\log_{10}|\Gamma_{in}| = 0$, Γ_L, CSOUT

$\log_{10}|\Gamma_{in}| = 0$,
$|\Gamma_{in}| = 1$

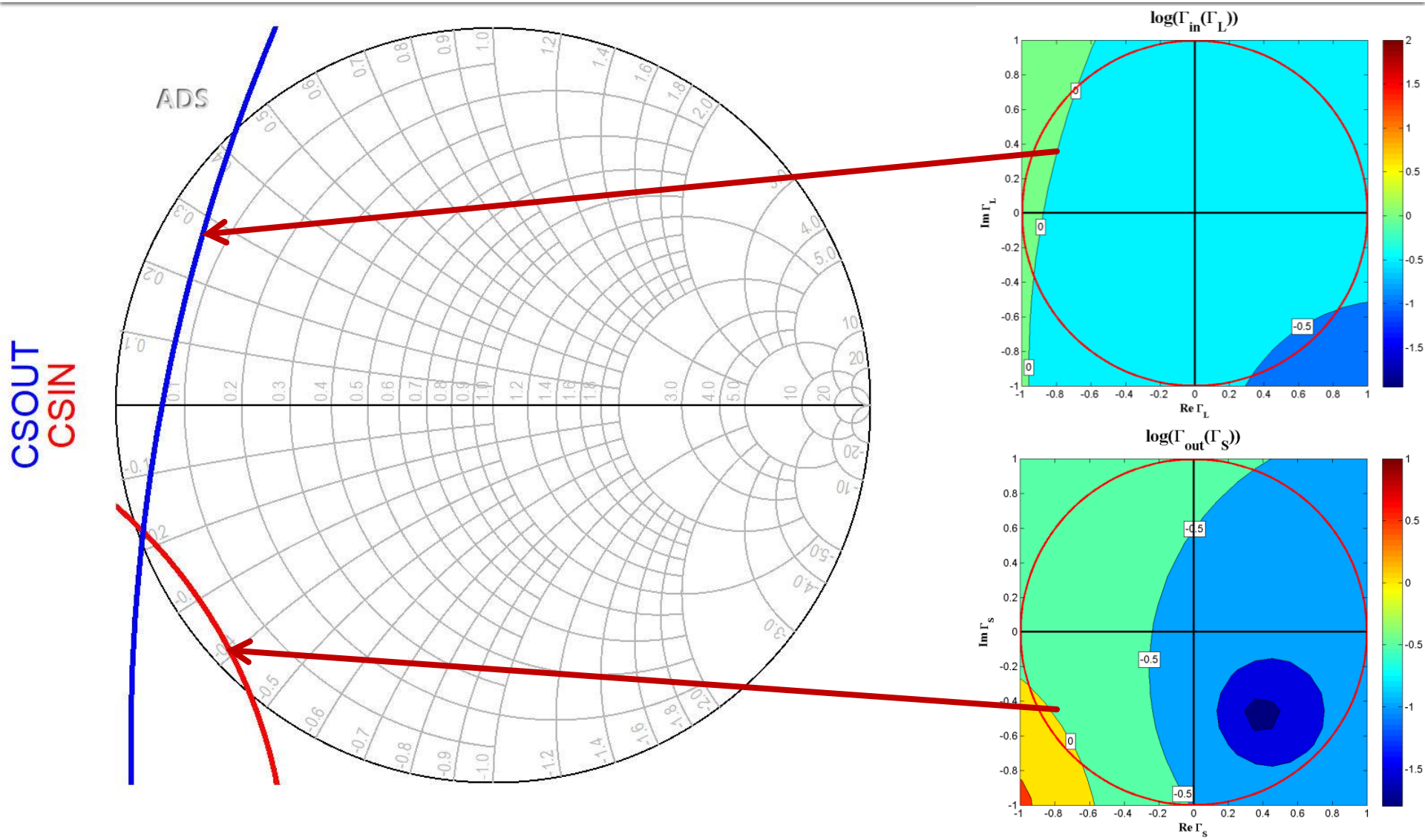


Contour lines of $\log_{10}|\Gamma_{out}|$

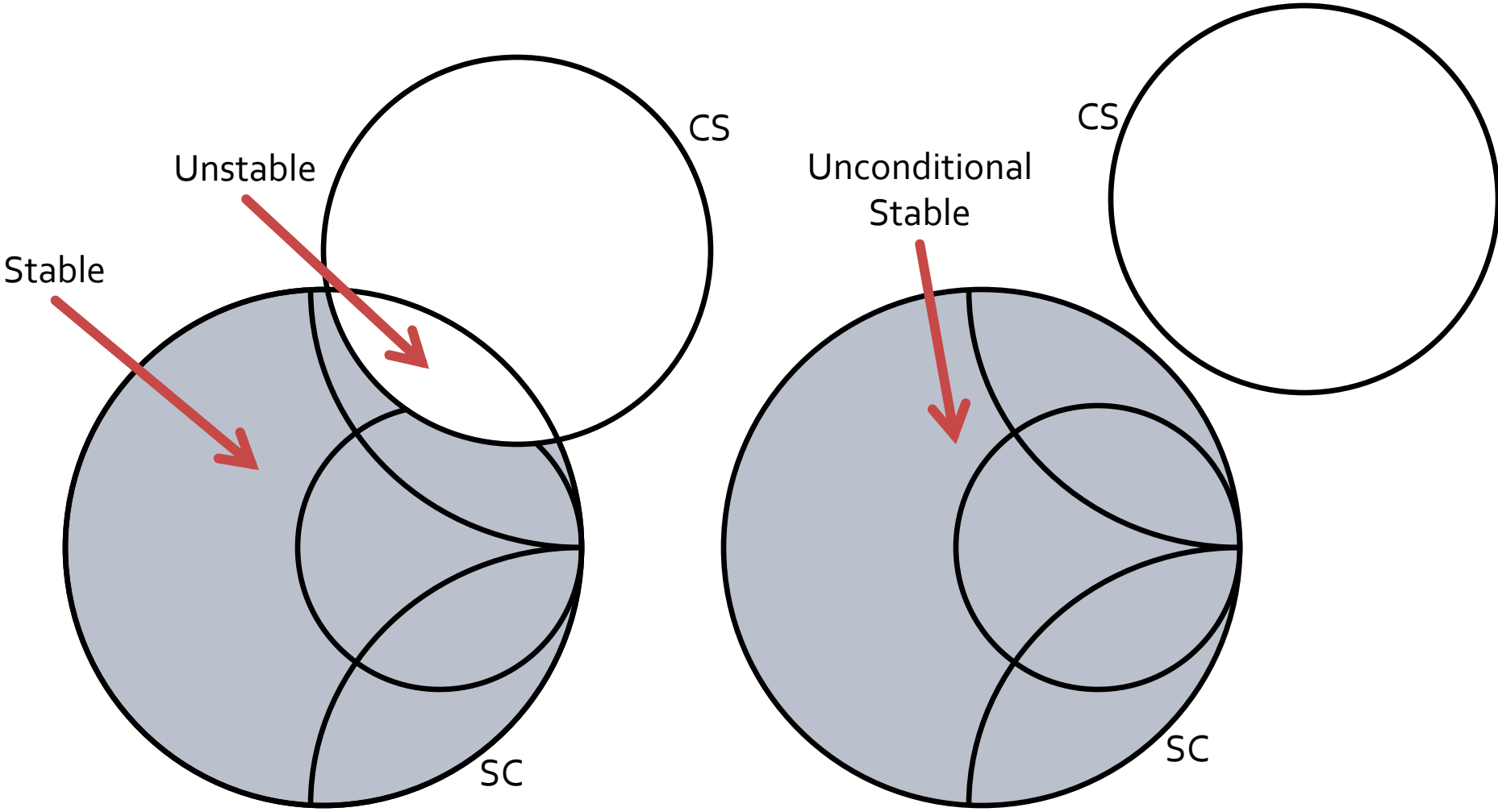
- $\log_{10}|\Gamma_{out}| = 0$, Γ_S, CSIN



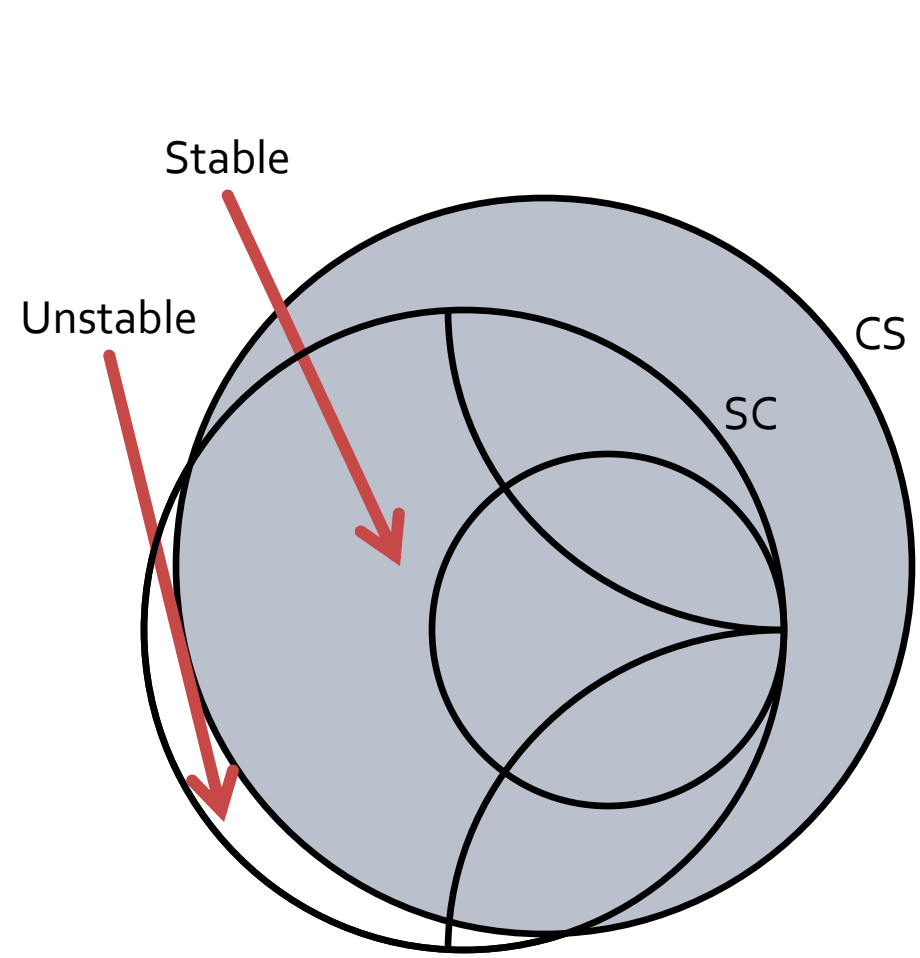
CSIN, CSOUT

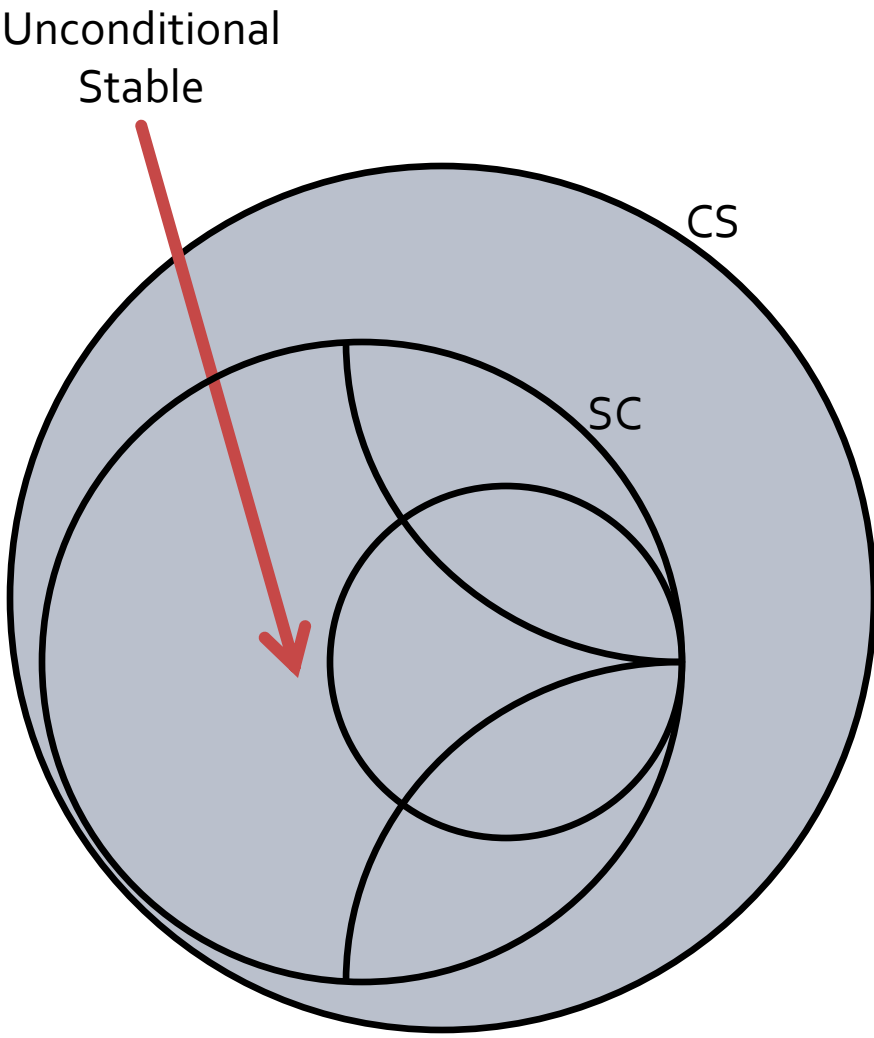


Several possible positioning

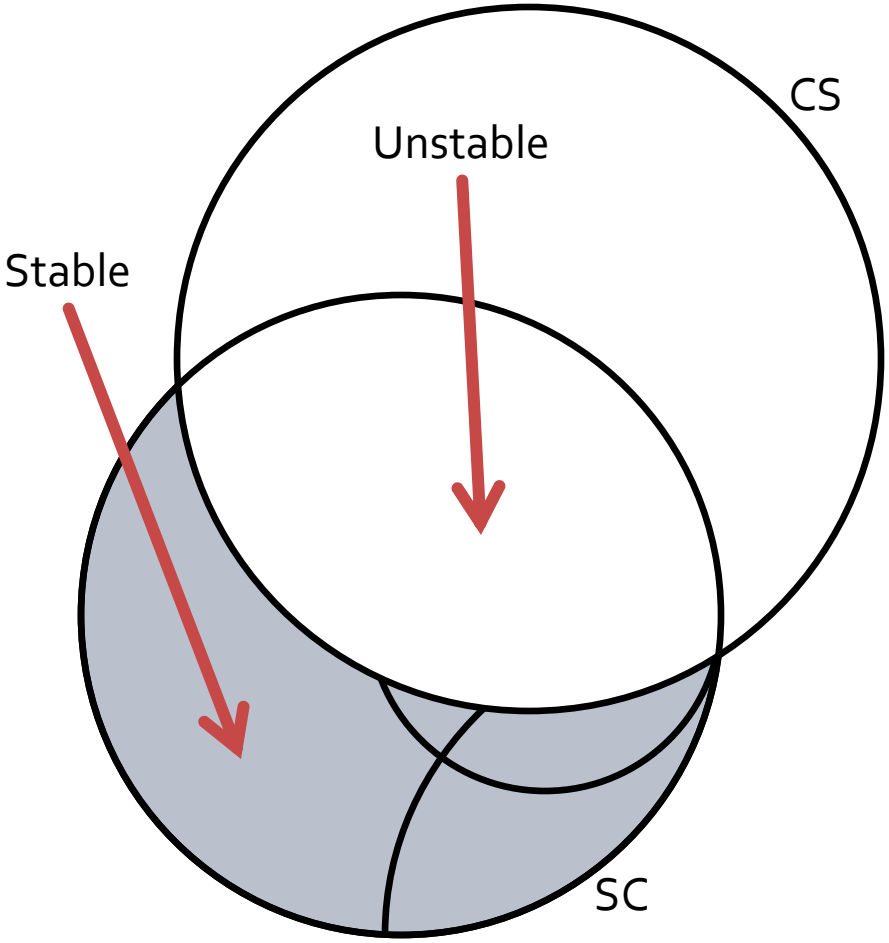


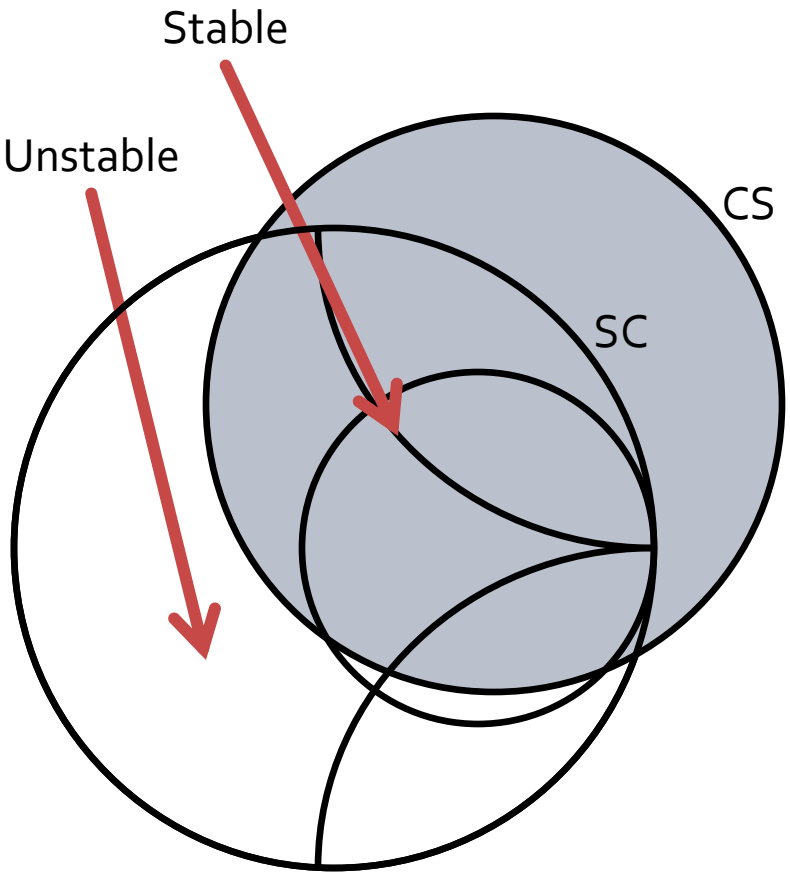
Several possible positioning





(Quite) Rare positioning





Stability

- **Unconditional stability:** the circuit is unconditionally stable if $|\Gamma_{in}|<1$ and $|\Gamma_{out}|<1$ for **any** passive impedance of the load/source
- **Conditional stability:** the circuit is conditionally stable if $|\Gamma_{in}|<1$ and $|\Gamma_{out}|<1$ only for **some** passive impedance of the load/source
  - passive impedance of the load/source <-> interior of the Smith Chart (radius 1 circle in the complex plane)

Unconditional stability

- The two-port is unconditionally stable if:
  - The stability circle is disjoint with the Smith Chart (exterior to the Chart) and the stable region is outside the circle
  - The stability circle encloses the entire Smith Chart and the stable region is inside the circle
- One mandatory condition for unconditional stability is |S11| < 1 (CSOUT) or |S22| < 1 (CSIN) – if in at least one point the two-port is not stable then it cannot be unconditionally stable
- Mathematically :

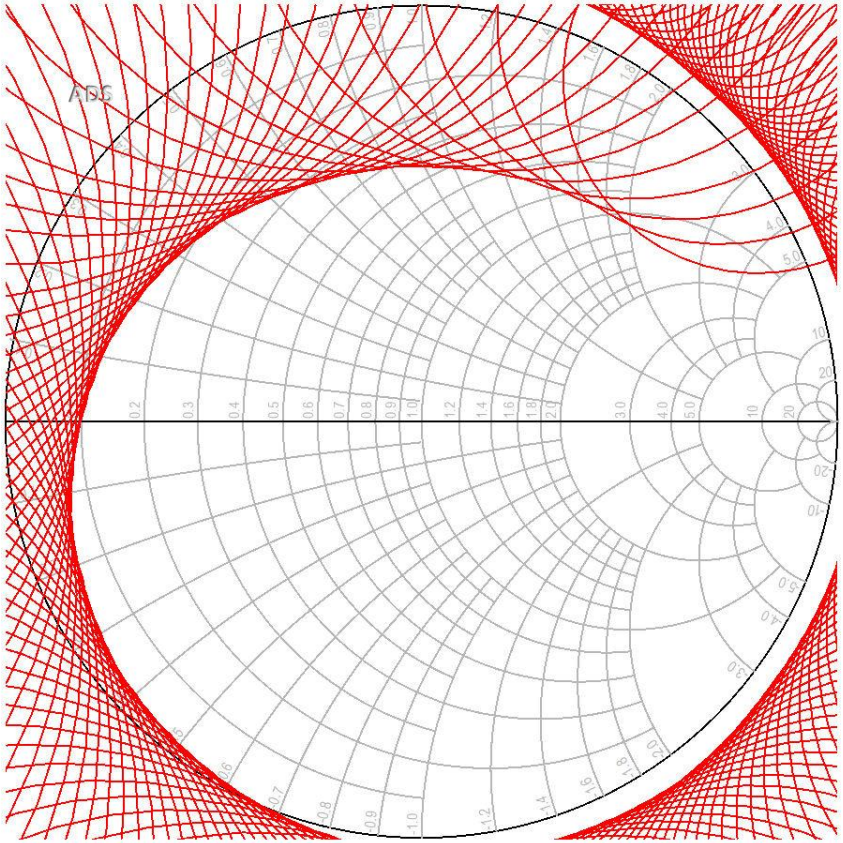
$$\begin{cases} \left||C_L|-R_L\right|>1 \\ |S_{11}|<1 \end{cases} \qquad \begin{cases} \left||C_S|-R_S\right|>1 \\ |S_{22}|<1 \end{cases}$$

Tests for Unconditional Stability

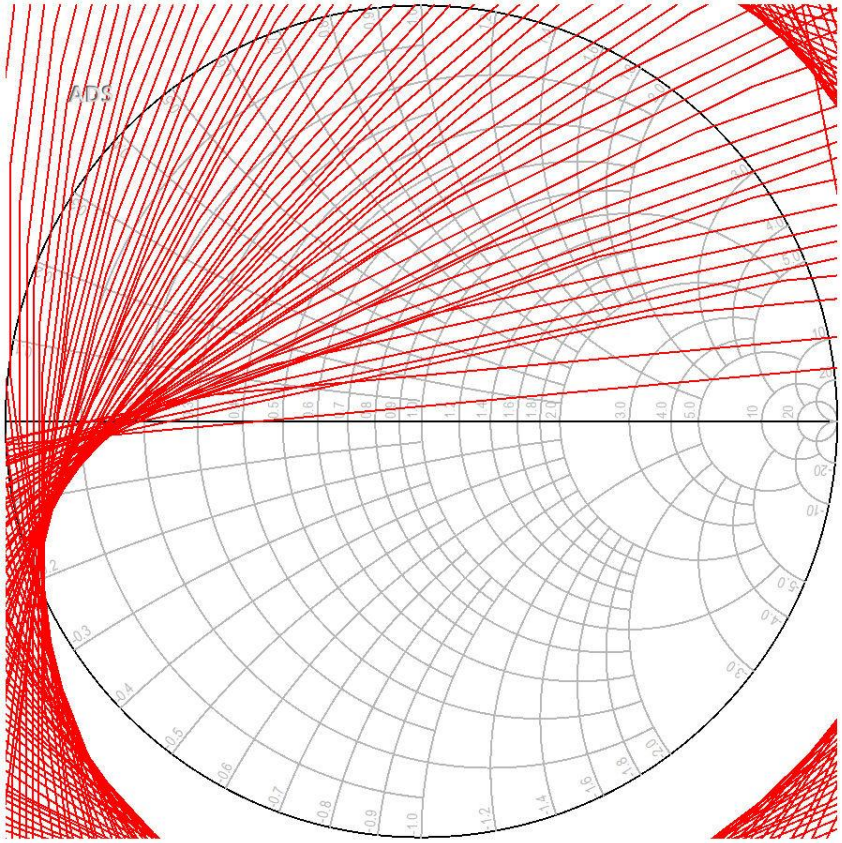
- Useful for wide frequency range analysis
- It is not enough to check the stability only at the operating frequencies
  - we must obtain stable operation for chosen Γ_L and Γ_S at **any** frequency

Circles in wide frequency range

CSIN



CSOUT



Rollet's condition

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|} \qquad \Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21}$$

- The two-port is **unconditionally stable** if:
- two conditions are simultaneously satisfied:
  - $K > 1$
  - $|\Delta| < 1$
- together with the implicit conditions:
  - $|S11| < 1$
  - $|S22| < 1$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|} > 1 \qquad |\Delta| = |S_{11} \cdot S_{22} - S_{12} \cdot S_{21}| < 1$$

μ Criterion

- Rollet's condition cannot be used to compare the relative stability of two or more devices because it involves constraints on two separate parameters, K and Δ

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - \Delta \cdot S_{11}^*| + |S_{12} \cdot S_{21}|} > 1$$

- The two-port is **unconditionally stable** if:
  - μ > 1
- together with the implicit conditions:
  - |S11| < 1
  - |S22| < 1
- In addition, it can be said that larger values of μ imply greater stability
  - μ is the distance from the center of the Smith Chart to the closest output stability circle

μ′ Criterion

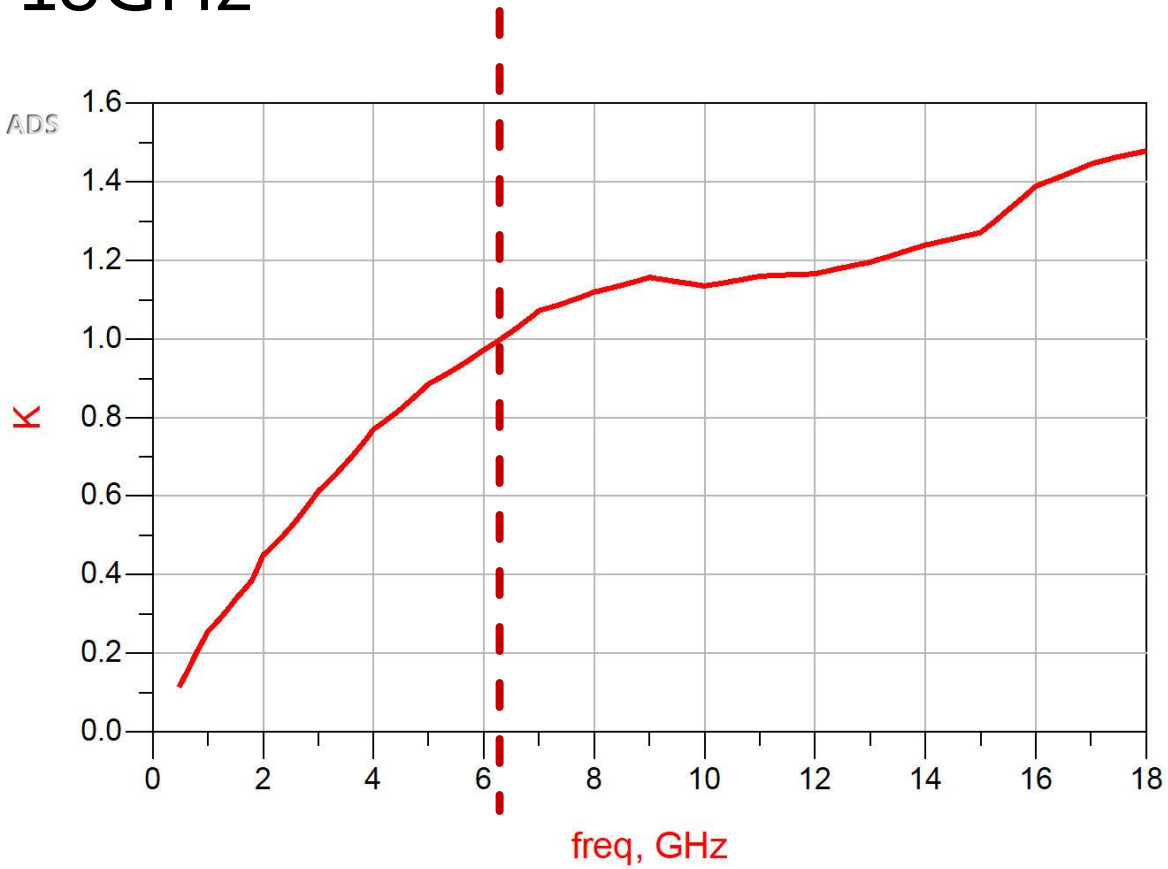
- Dual parameter to μ, determined in relation to the input stability circles

$$\mu' = \frac{1 - |S_{22}|^2}{|S_{11} - \Delta \cdot S_{22}^*| + |S_{12} \cdot S_{21}|} > 1$$

- The two-port is **unconditionally stable** if:
  - μ′ > 1
- together with the implicit conditions:
  - |S11| < 1
  - |S22| < 1
- In addition, it can be said that larger values of μ′ imply greater stability
  - μ′ is the distance from the center of the Smith Chart to the closest input stability circle

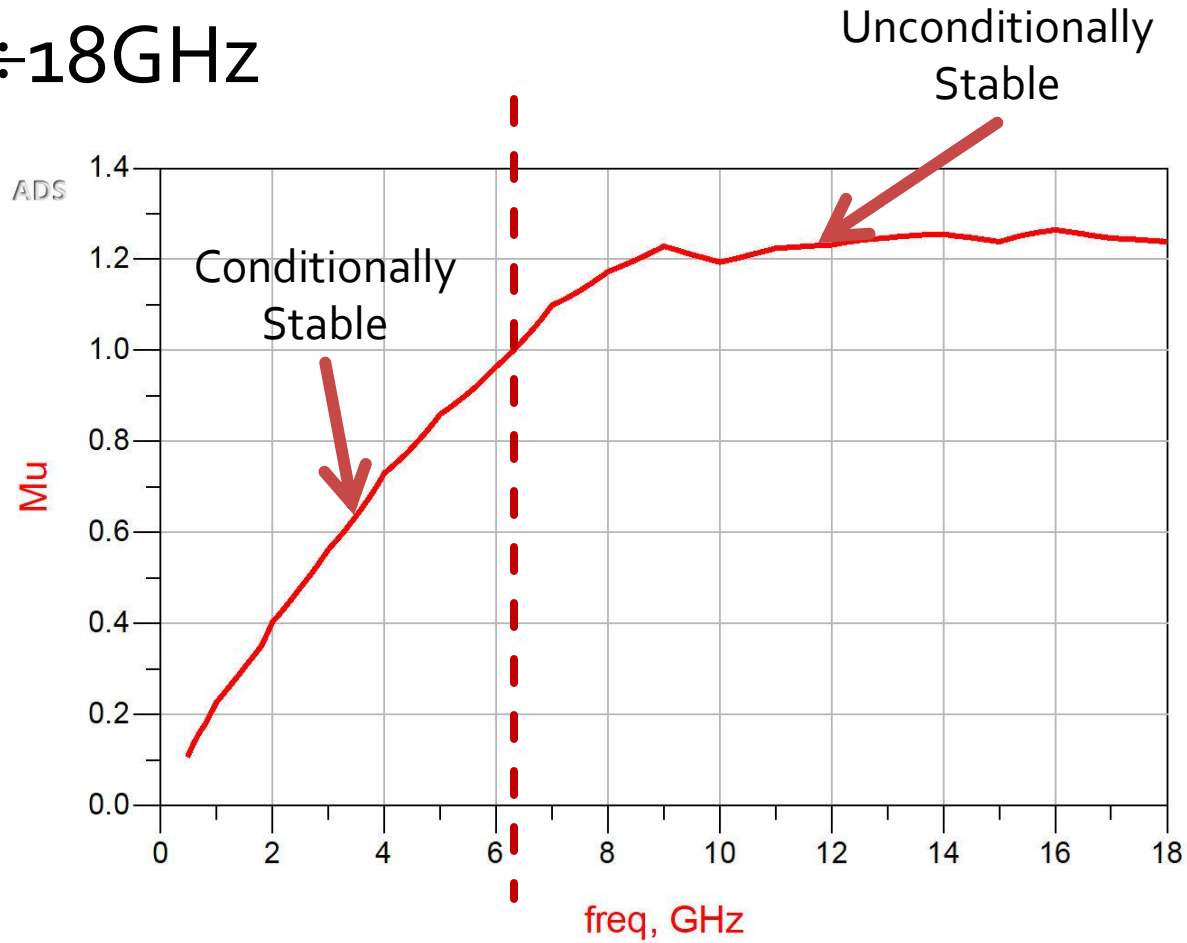
Rollet's condition

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



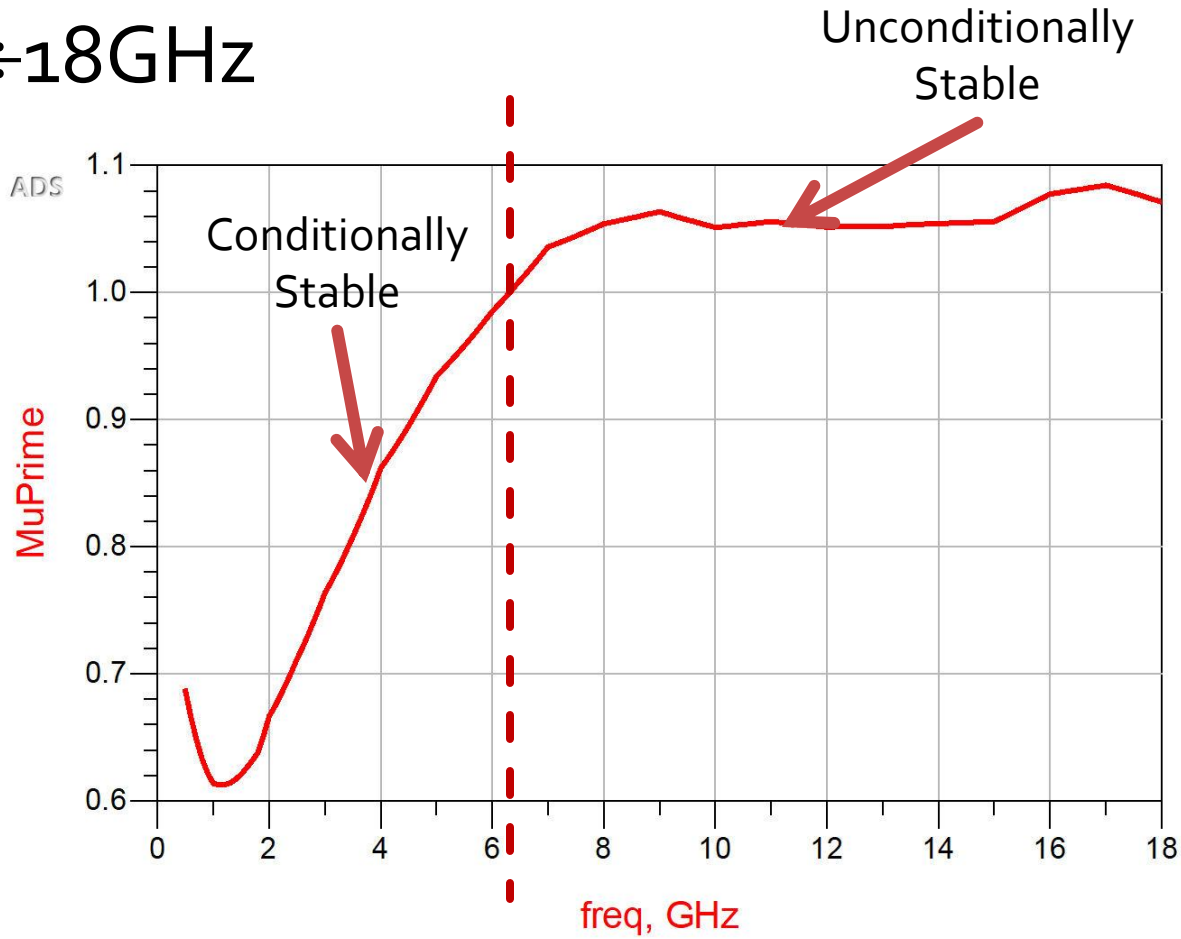
μ Criterion

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



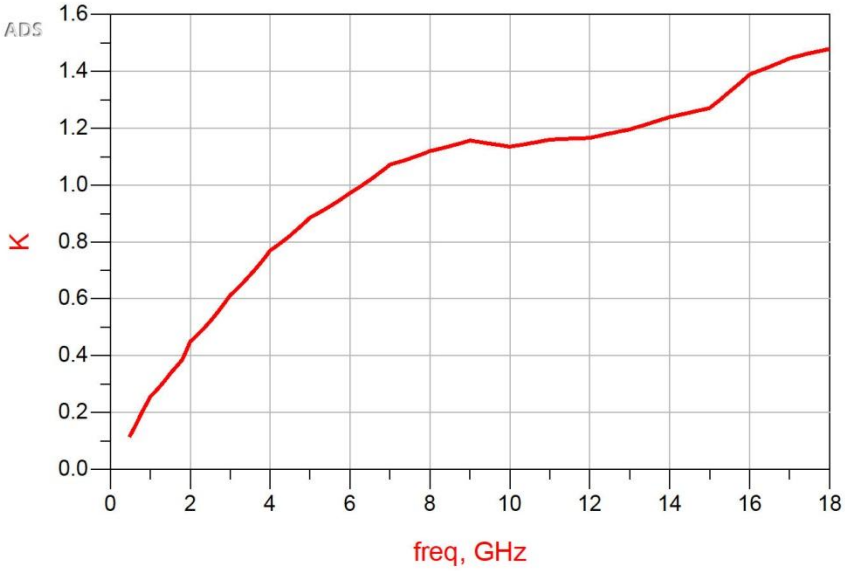
μ' Criterion

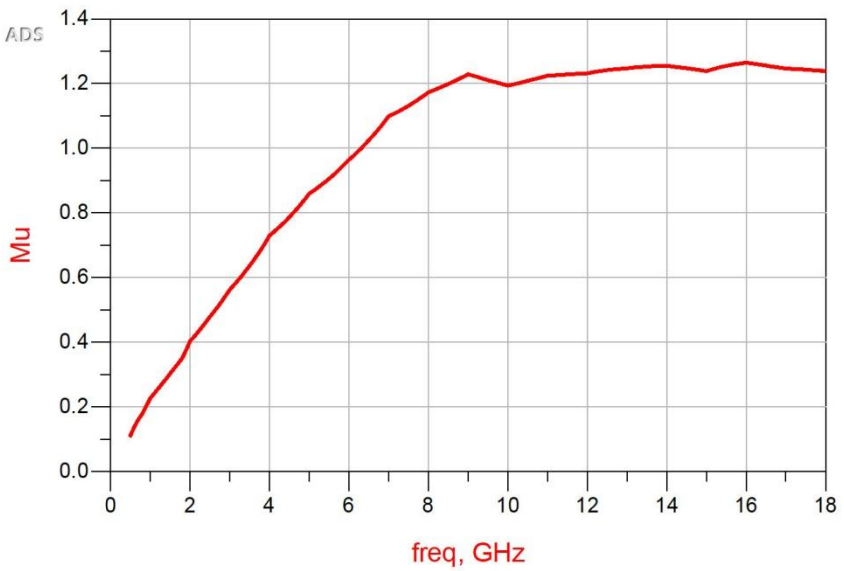
- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



Stability

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz
- unconditionally stable for f > 6.31GHz

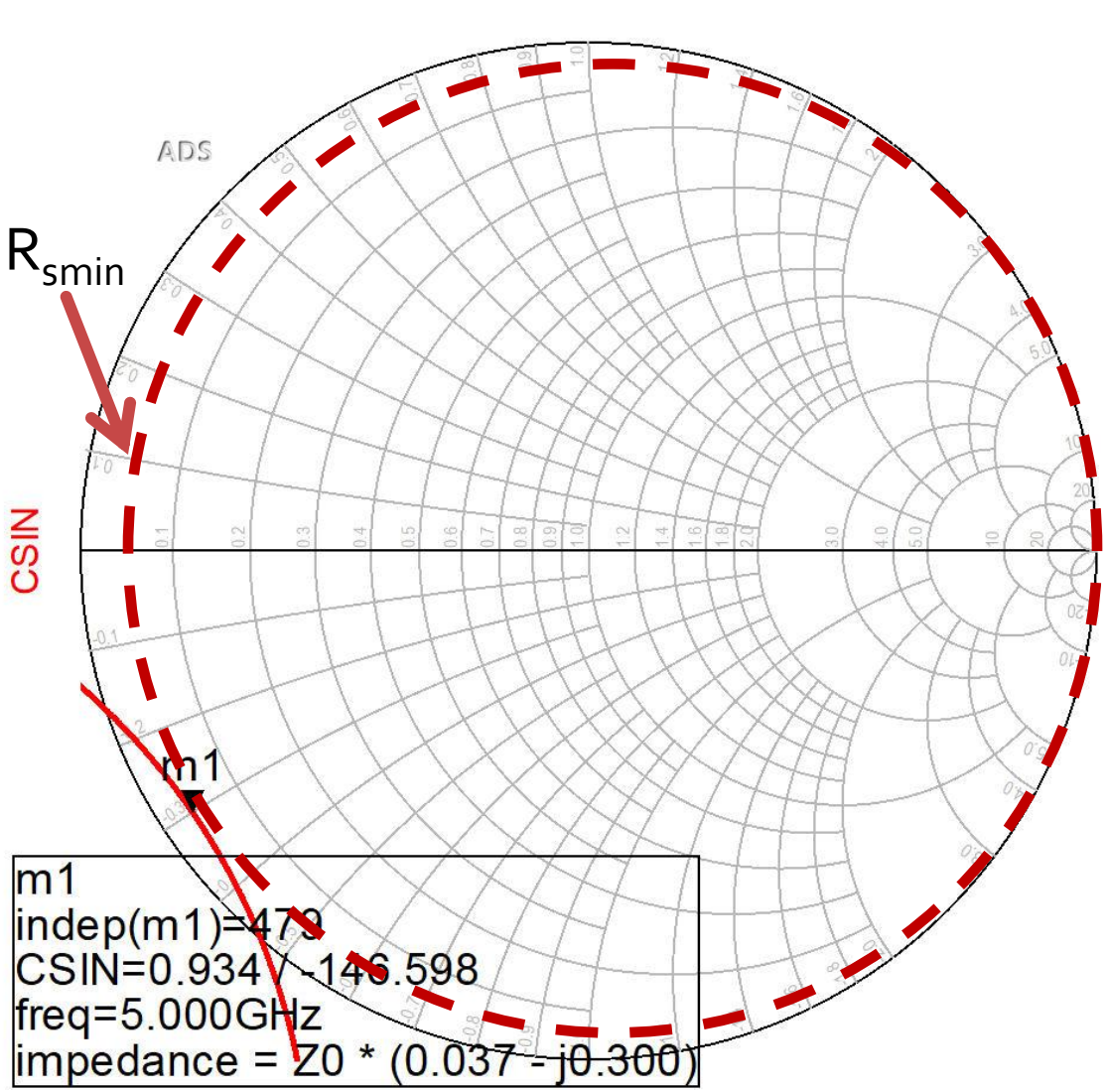


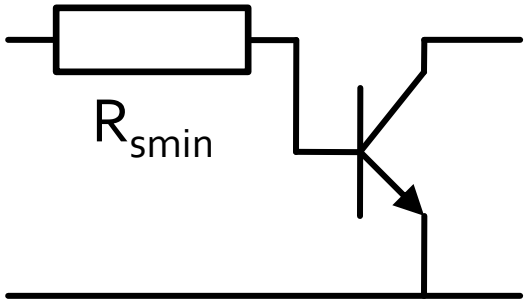


Stabilization of two-port

- Unconditional stability in a wide frequency range has some important advantages
  - Ex: We can use ATF 34143 to design a (conditionally) stable amplifier at 5GHz, but this design is useless if the amplifier oscillates at 500MHz ($\mu \approx 0.1$)
- **The minimal requirement** when working with conditionally stable devices is to **check stability** at several frequencies over the operating bandwidth and outside the bandwidth
- Unconditional stability can be forced by inserting series/shunt resistors at two-port's input/output (with loss of gain!)

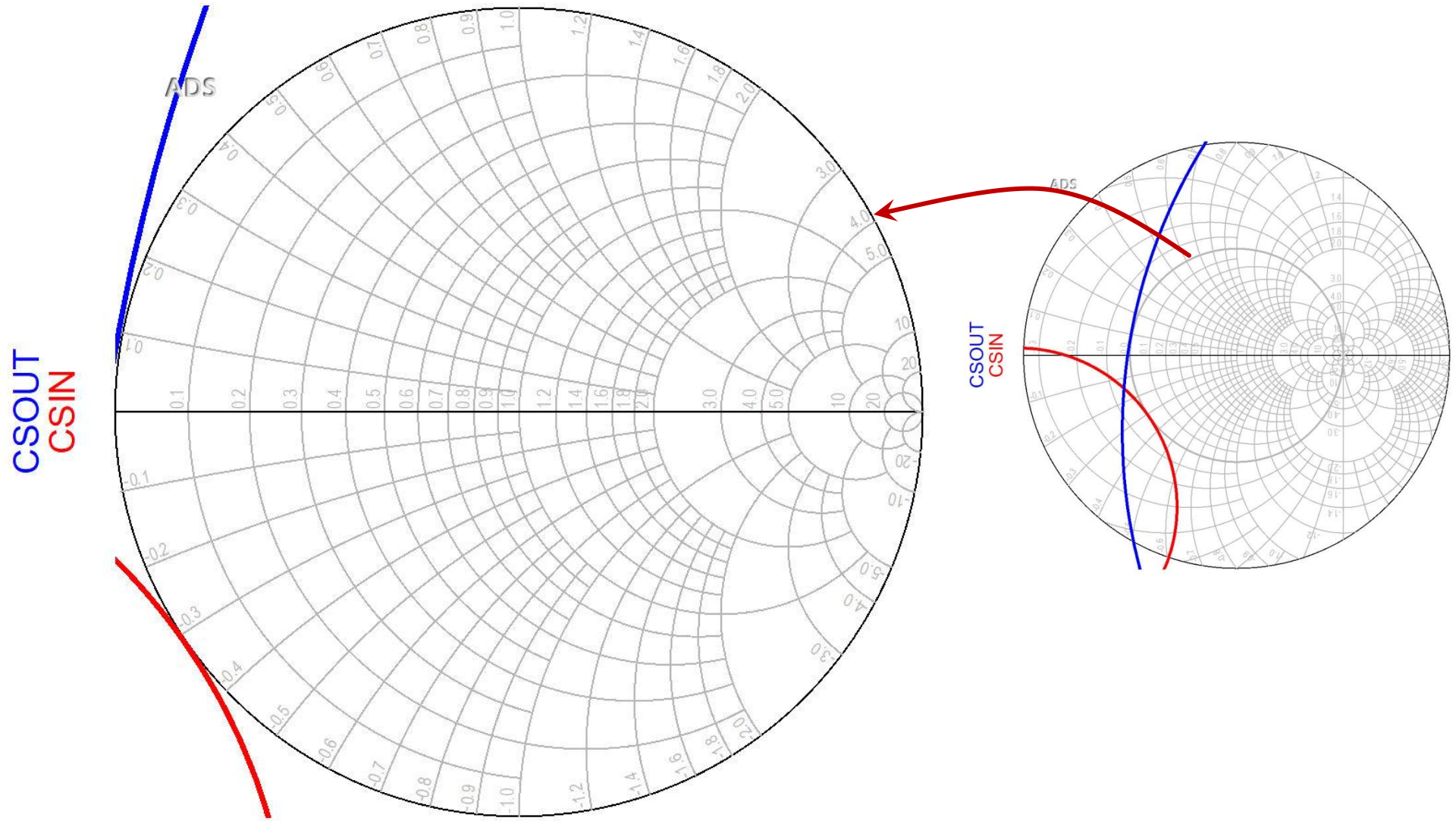
Input series resistor





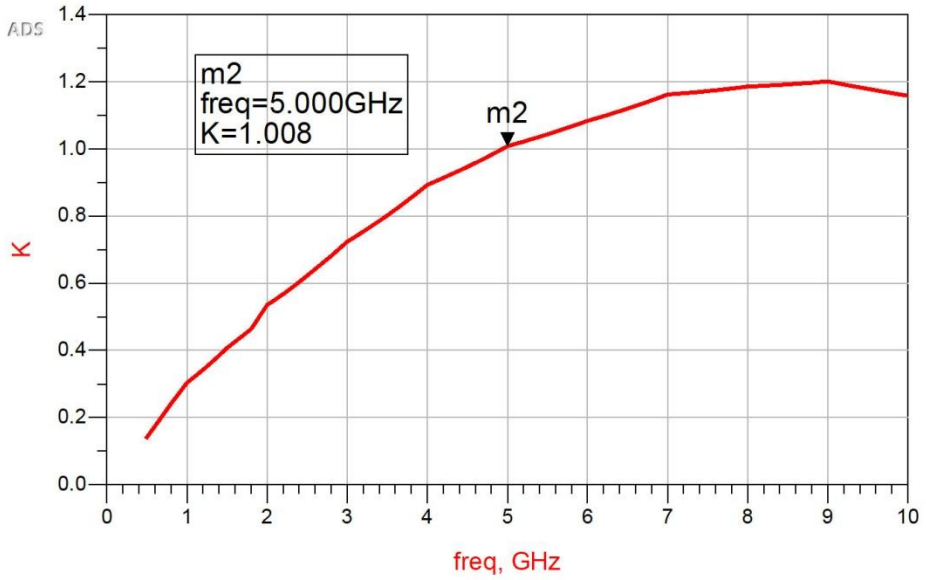
$$R_{s\min} = 0.037 \cdot 50\Omega = 1.85\Omega$$

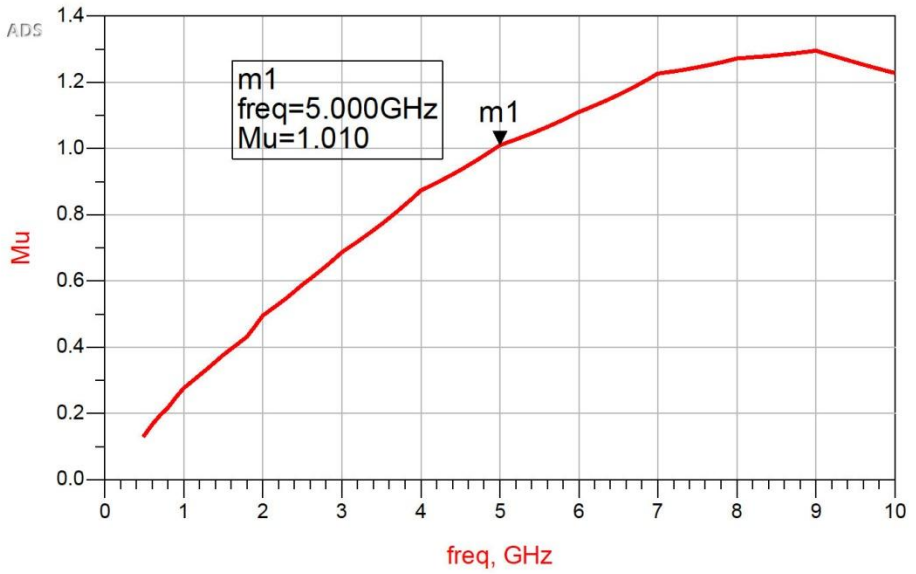
ADS, Rs = 2Ω



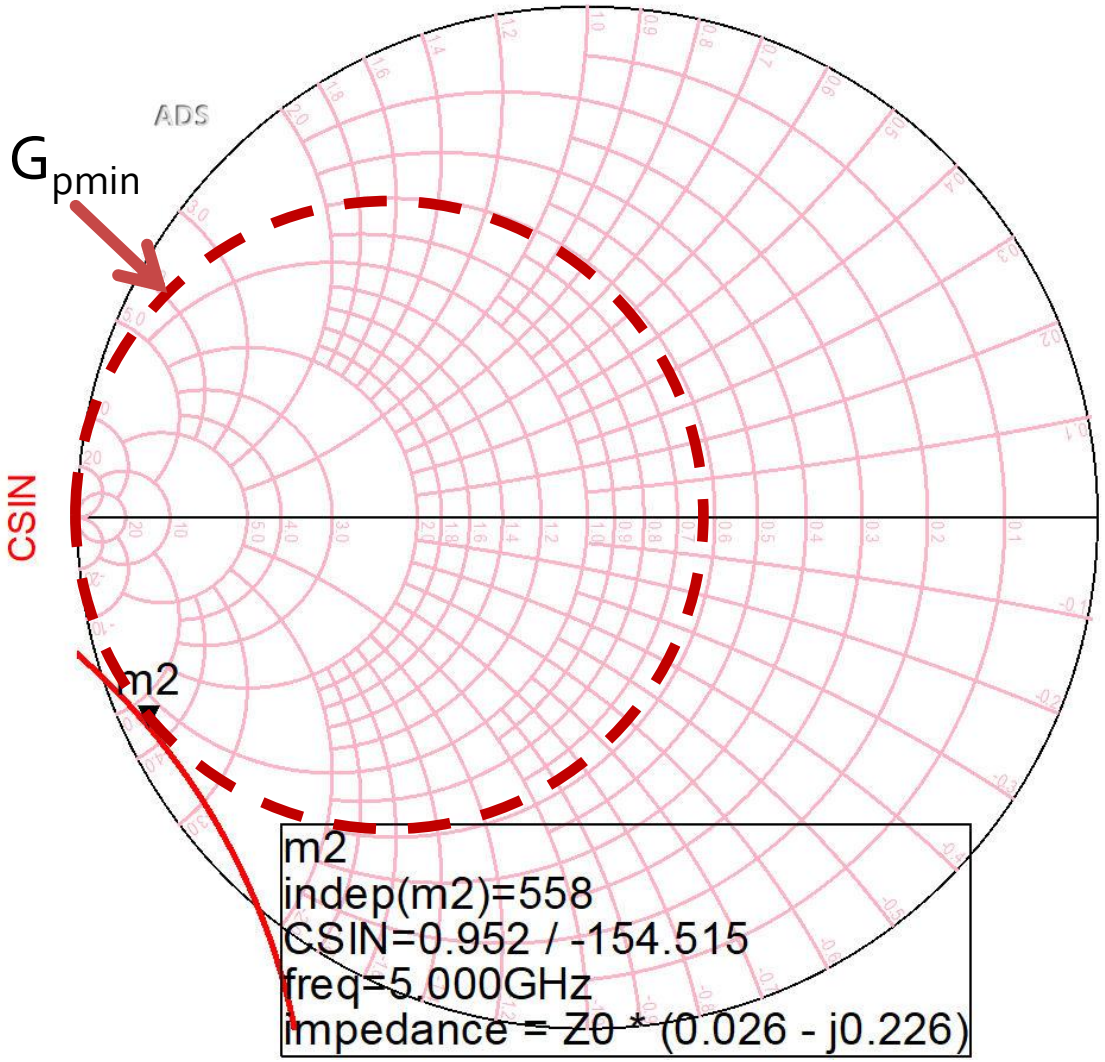
Input series resistor

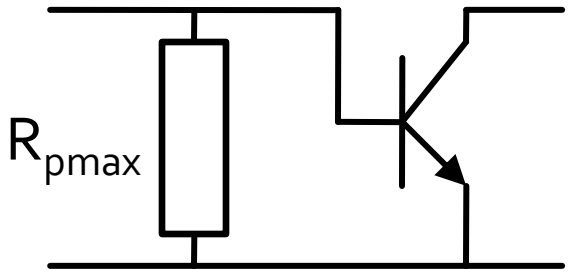
- Rs = 2Ω
- K = 1.008, MAG = 13.694dB @ 5GHz
  - no stabilization, K = 0.886, MAG = 14.248dB @ 5GHz





Input shunt resistor



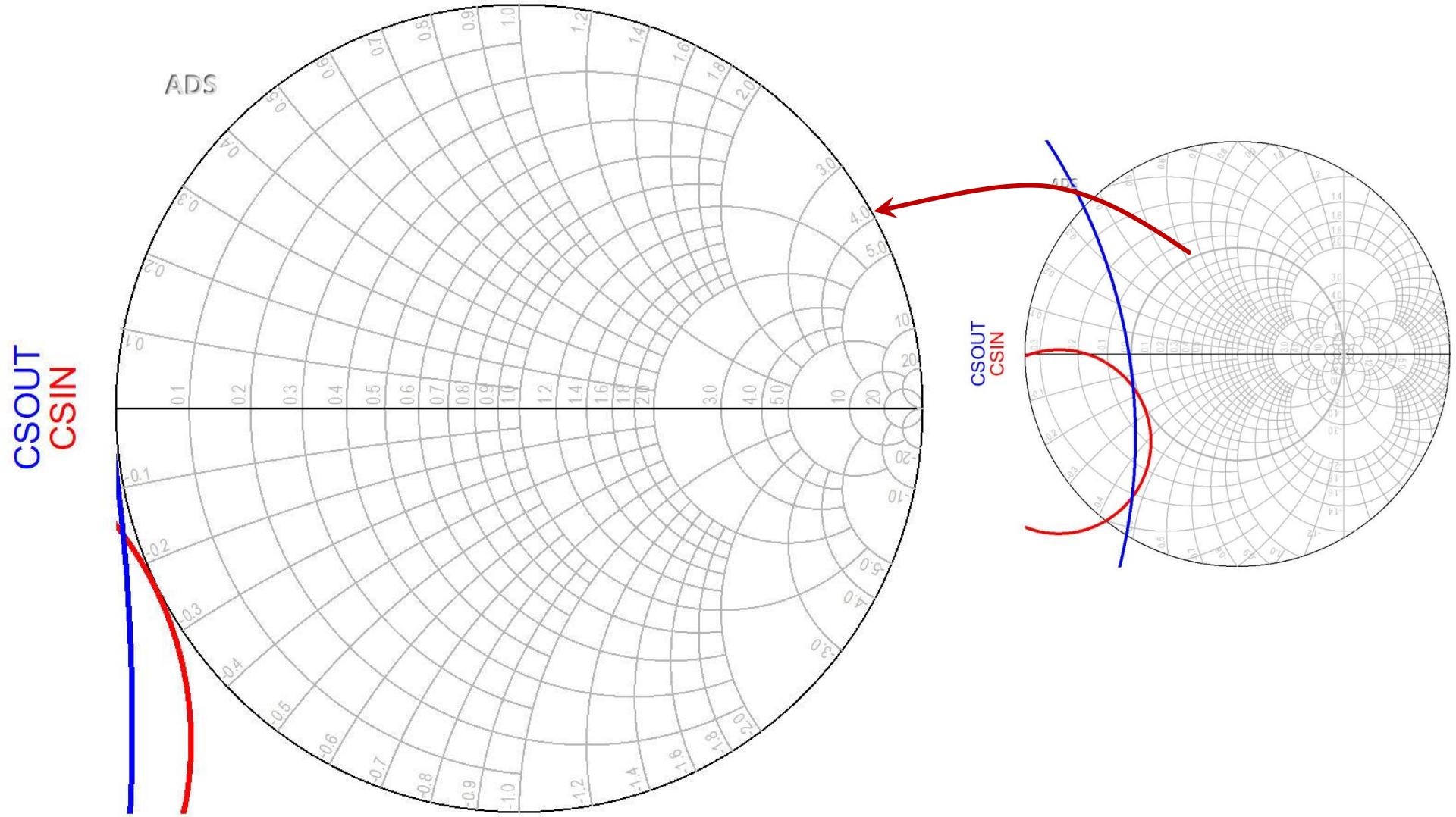


$$R_{p\max} = \frac{1}{G_{p\min}}$$

$$\frac{1}{0.026 - j \cdot 0.226} = 0.502 + j \cdot 4.367$$

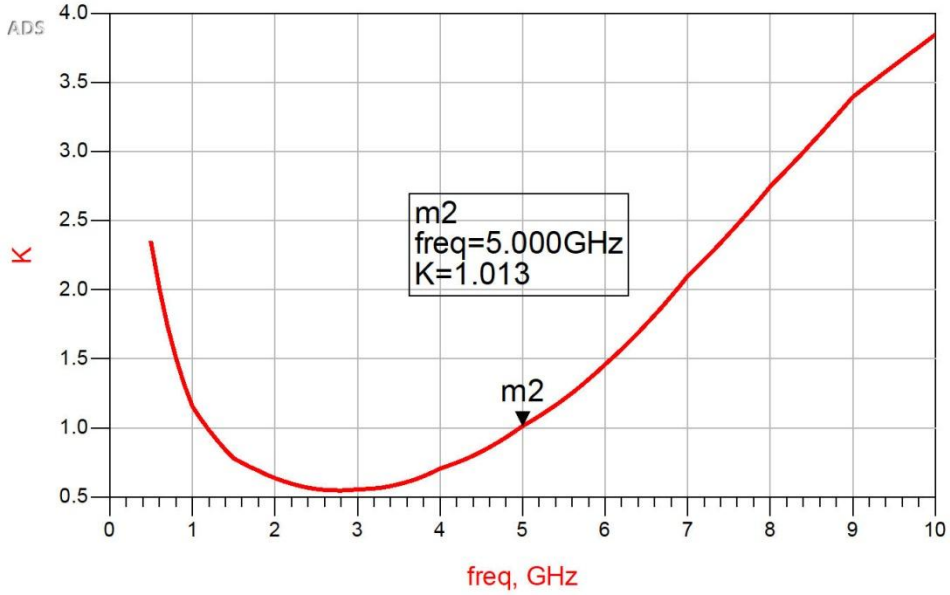
$$R_{p\max} = \frac{50\Omega}{0.502} = 99.6\Omega$$

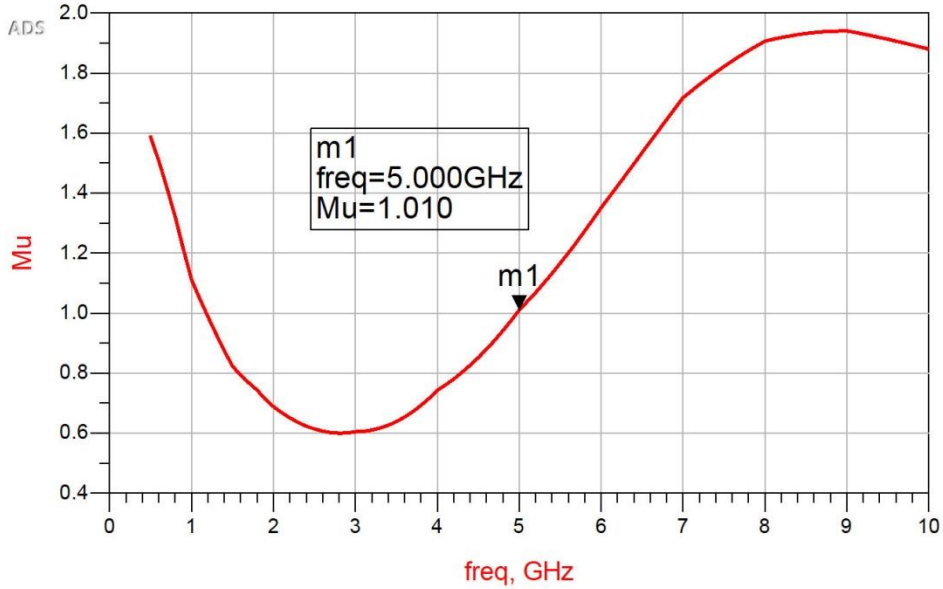
ADS, Rp = 90Ω



Input shunt resistor

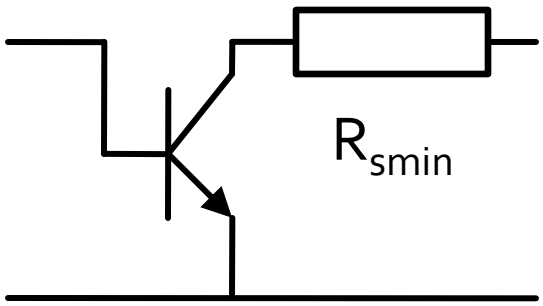
- Rp = 90Ω
- K = 1.013, MAG = 13.561dB @ 5GHz
  - no stabilization, K = 0.886, MAG = 14.248dB @ 5GHz

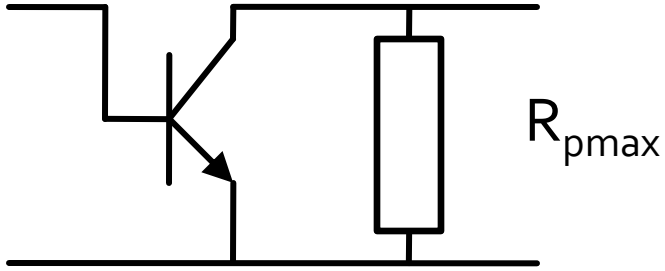




Output series/shunt resistor

- The procedure can be applied similarly at the output (finding g/r circles tangent to CSOUT)
- From previous examples, resistive loading at the input has a positive effect over output stability and vice versa (resistive loading at the output, effect over input stability)

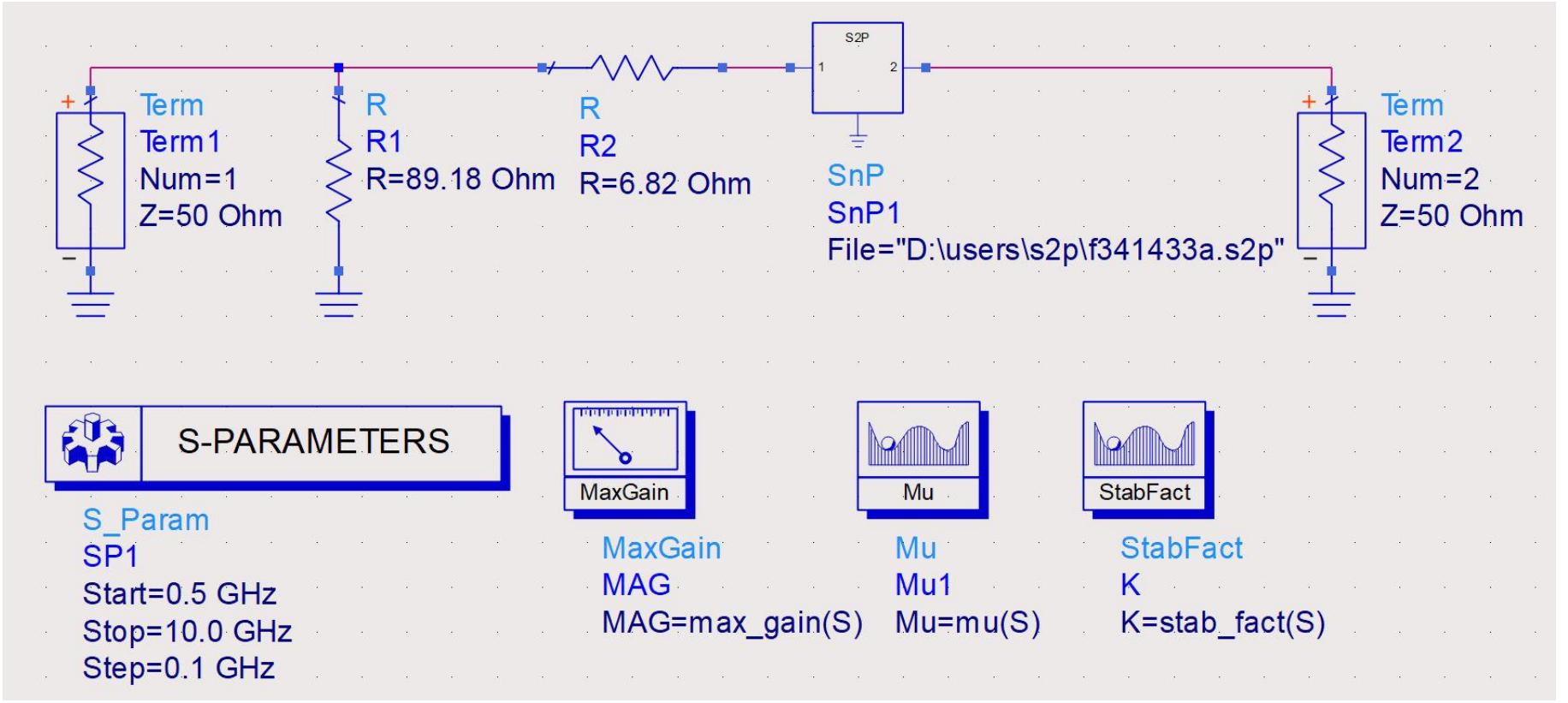




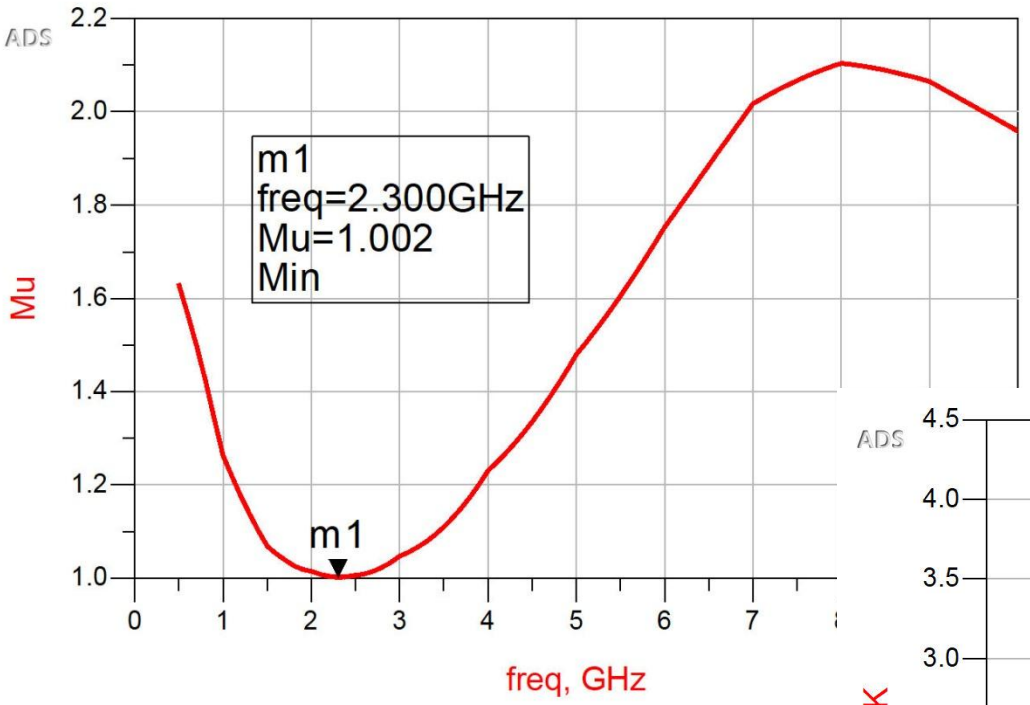
Stabilization of two-port

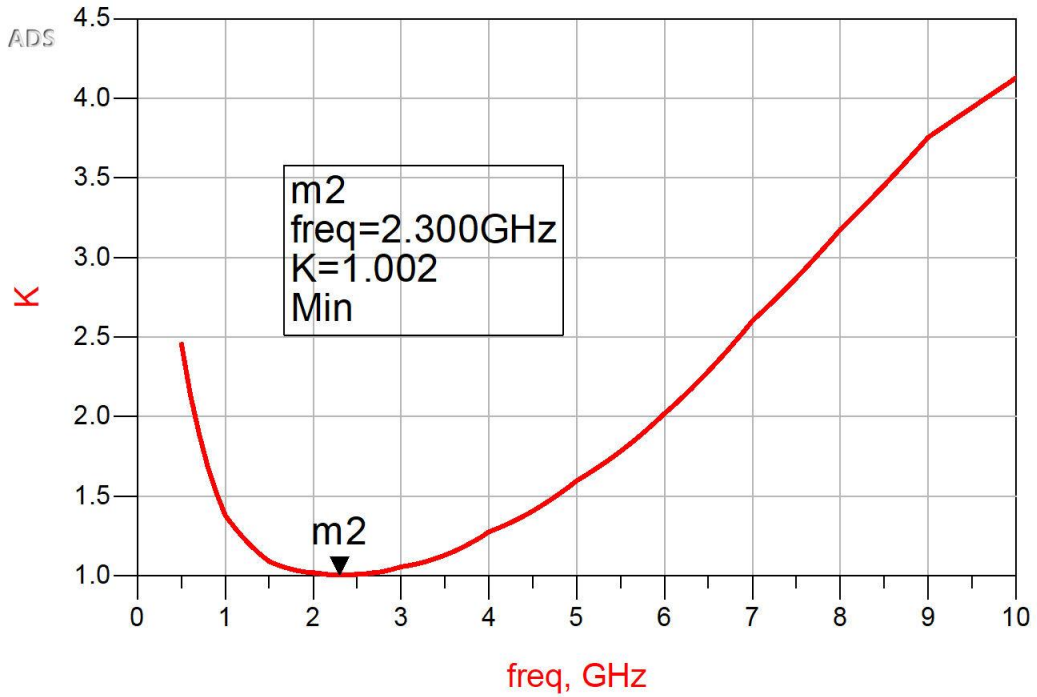
- Negative effect over the power gain
  - we must check MAG/MSG while designing resistive loading
- Negative effect over the noise (debated next)
- We can choose one of the 4 possibilities or a combination which offers better results (depending on transistor, application etc.)
- We can use frequency selective loading
  - Ex: RL, RC circuits which sacrifice performance only when needed to improve stability and have no effect at frequencies where the device is already stable
- It might be possible (and should be checked) that stability is improved as an effect of parasitic elements of biasing circuits (bypass capacitors and RF chokes)

Stabilization of two-port

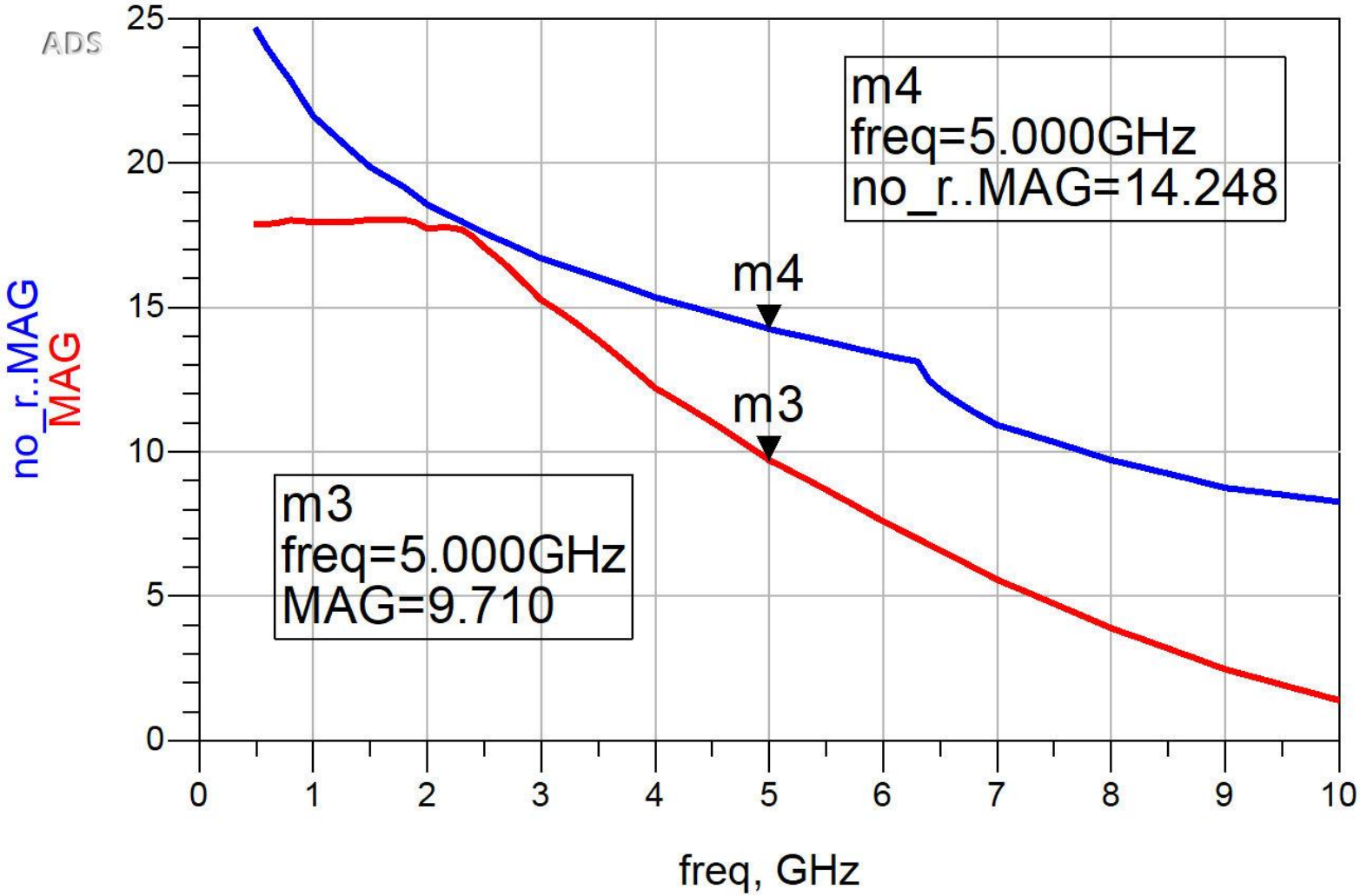


Stabilization of two-port

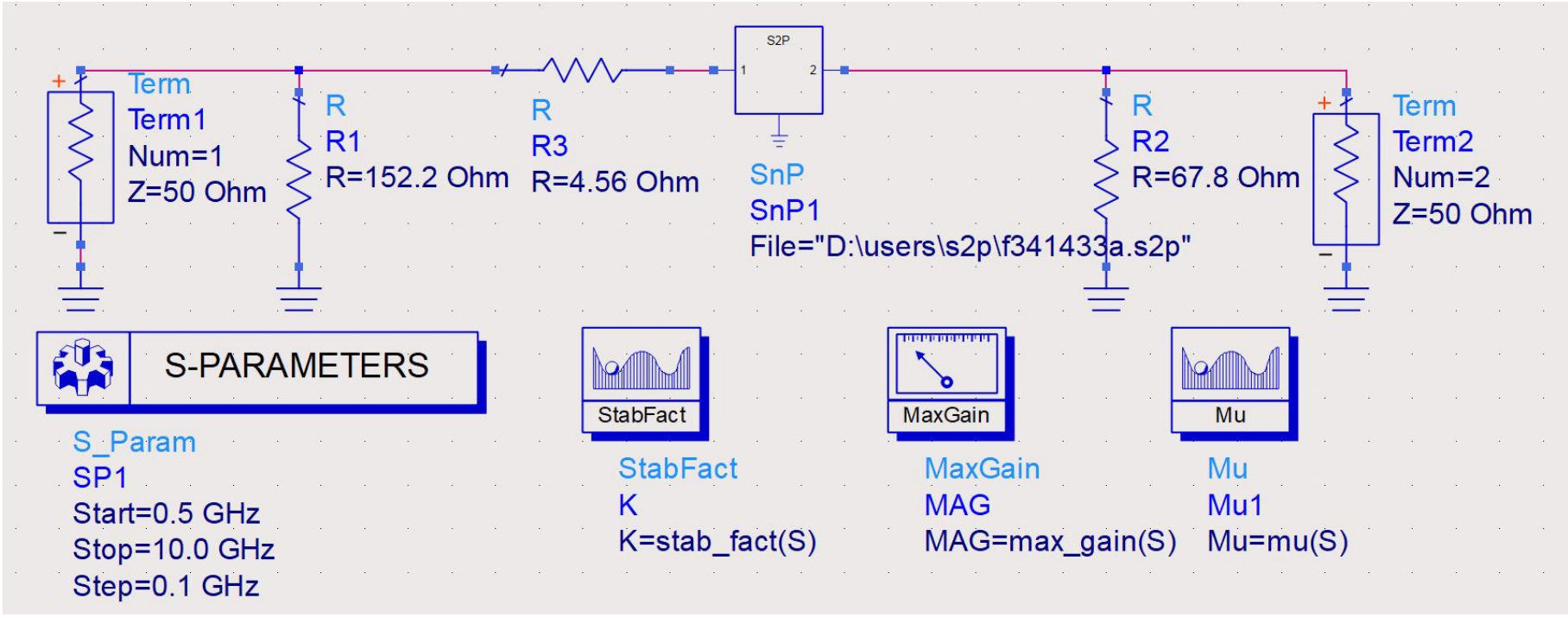




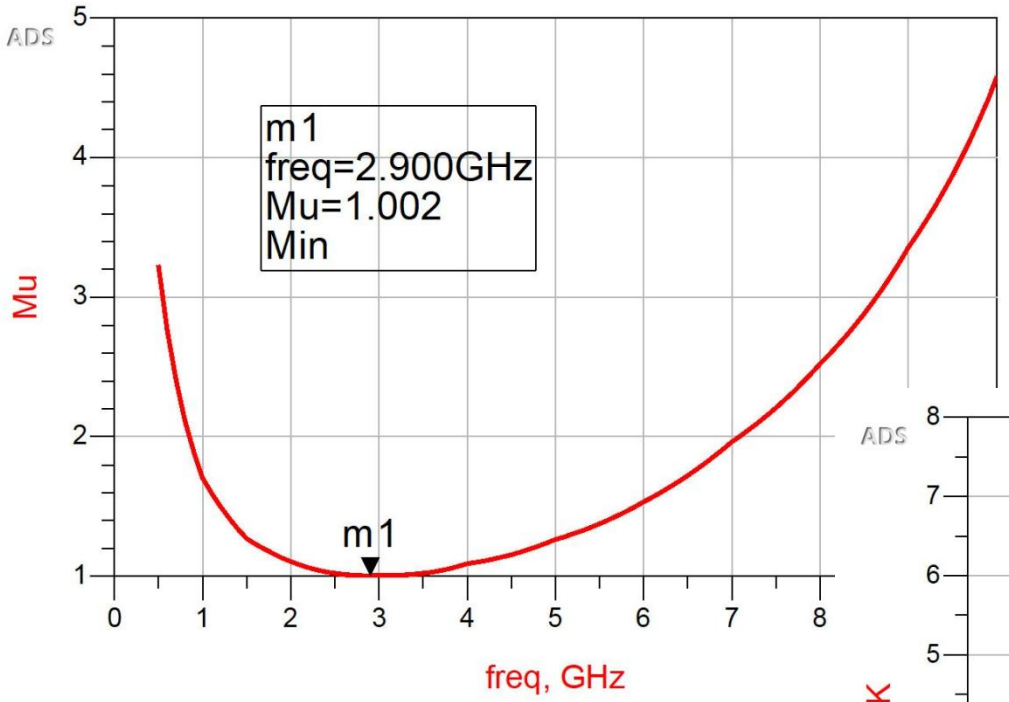
Stabilization of two-port

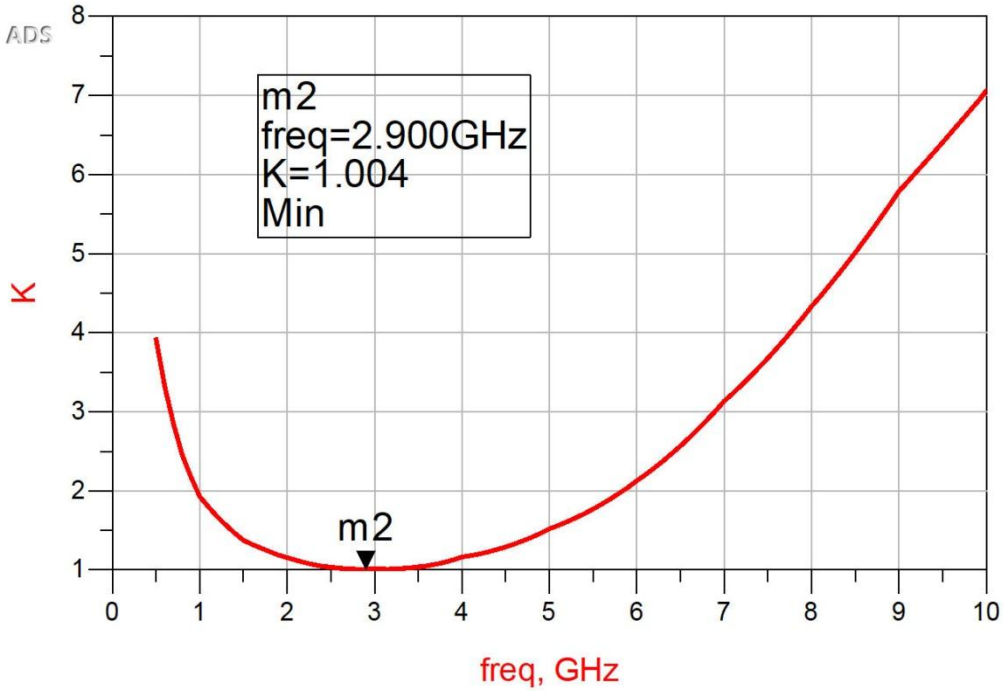


Stabilization of two-port

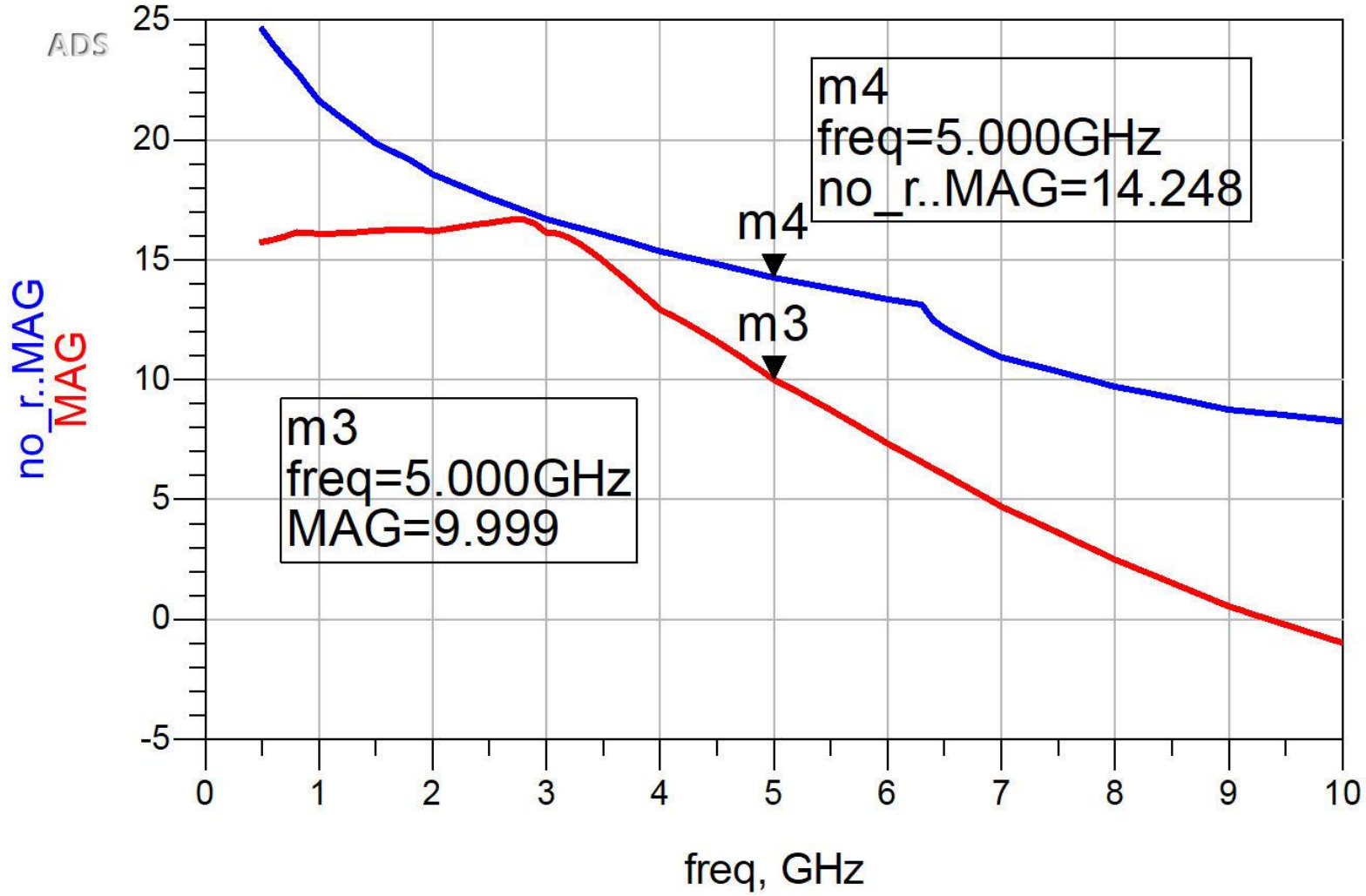


Stabilization of two-port





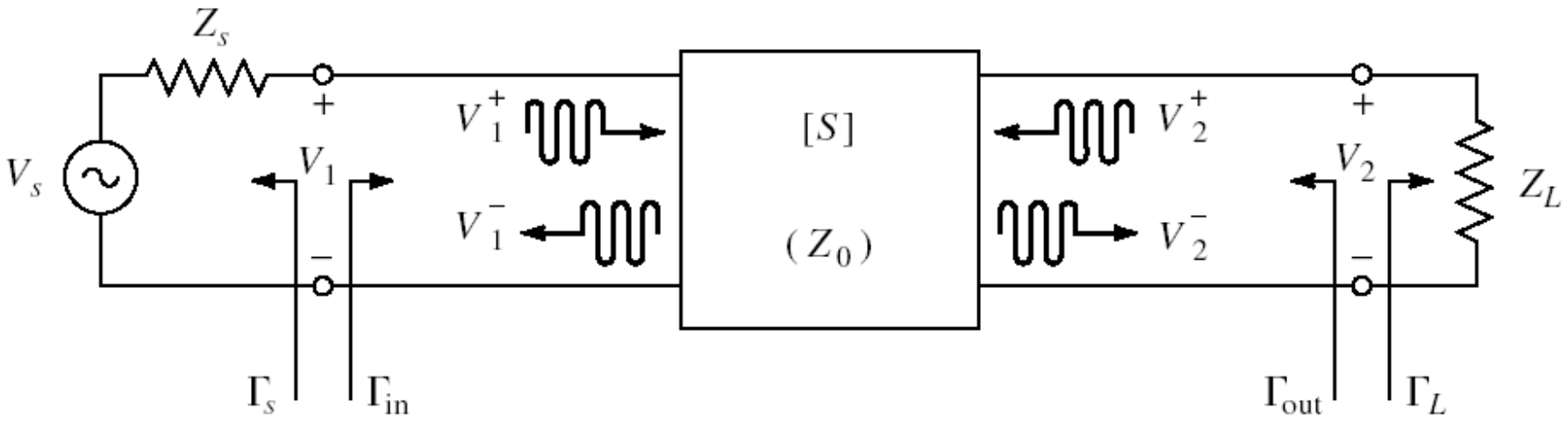
Stabilization of two-port



Microwave Amplifiers

Power Gain of Microwave Amplifiers

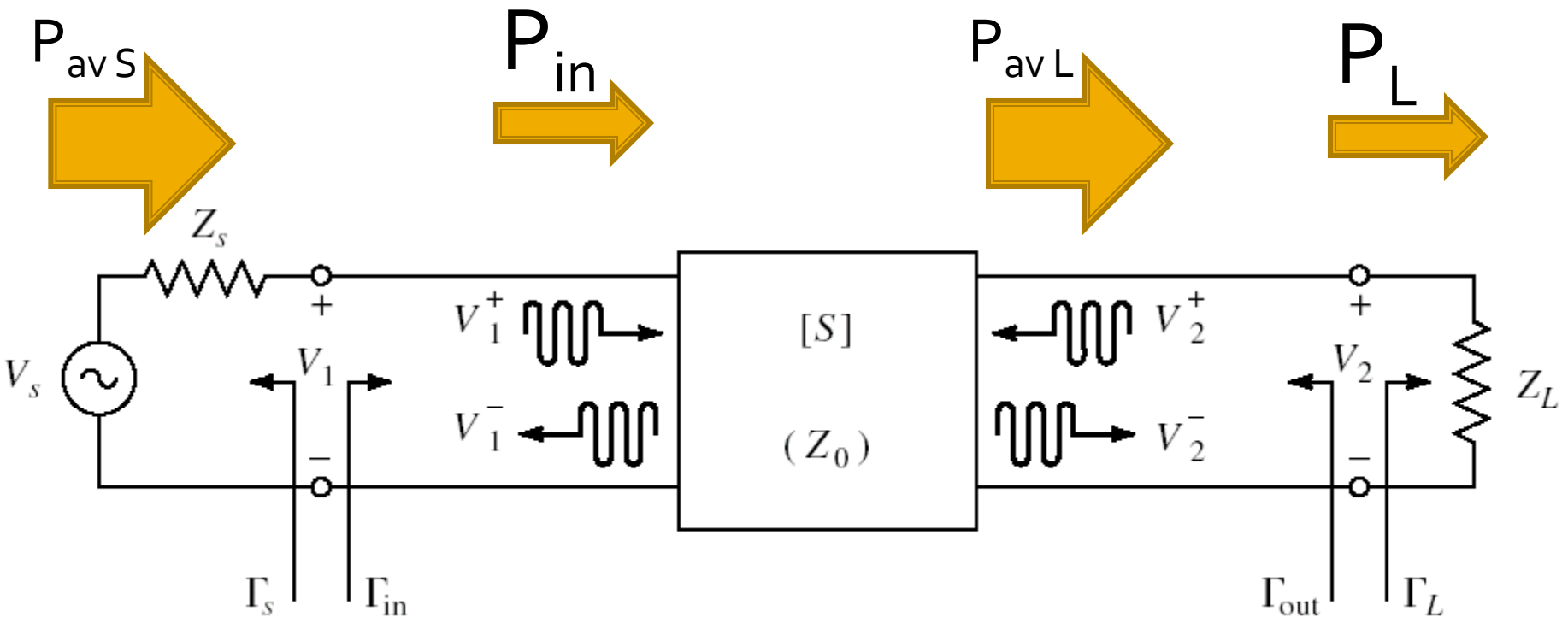
Amplifier as two-port



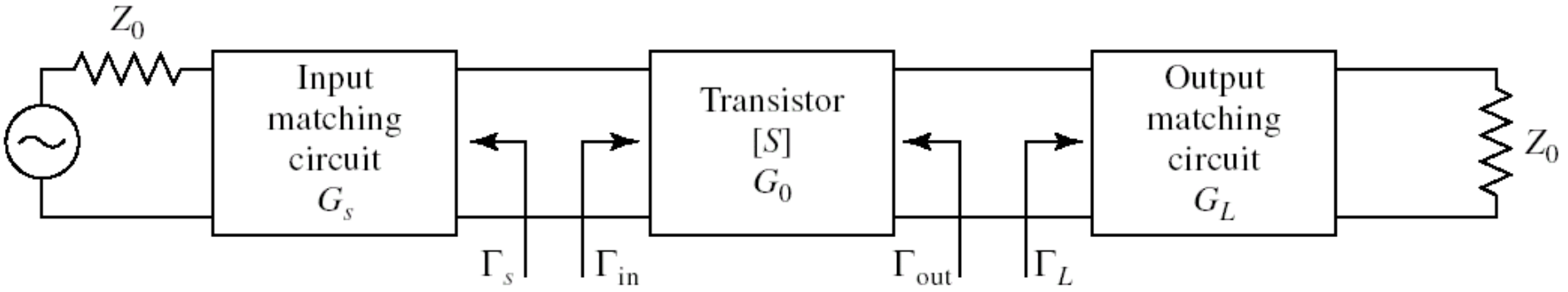
- For an amplifier two-port we are interested in:
  - stability
  - **power gain**
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Power / Matching

- Two ports in which matching influences the power transfer



Design for Maximum Gain



- Maximum power gain (complex conjugate matching):

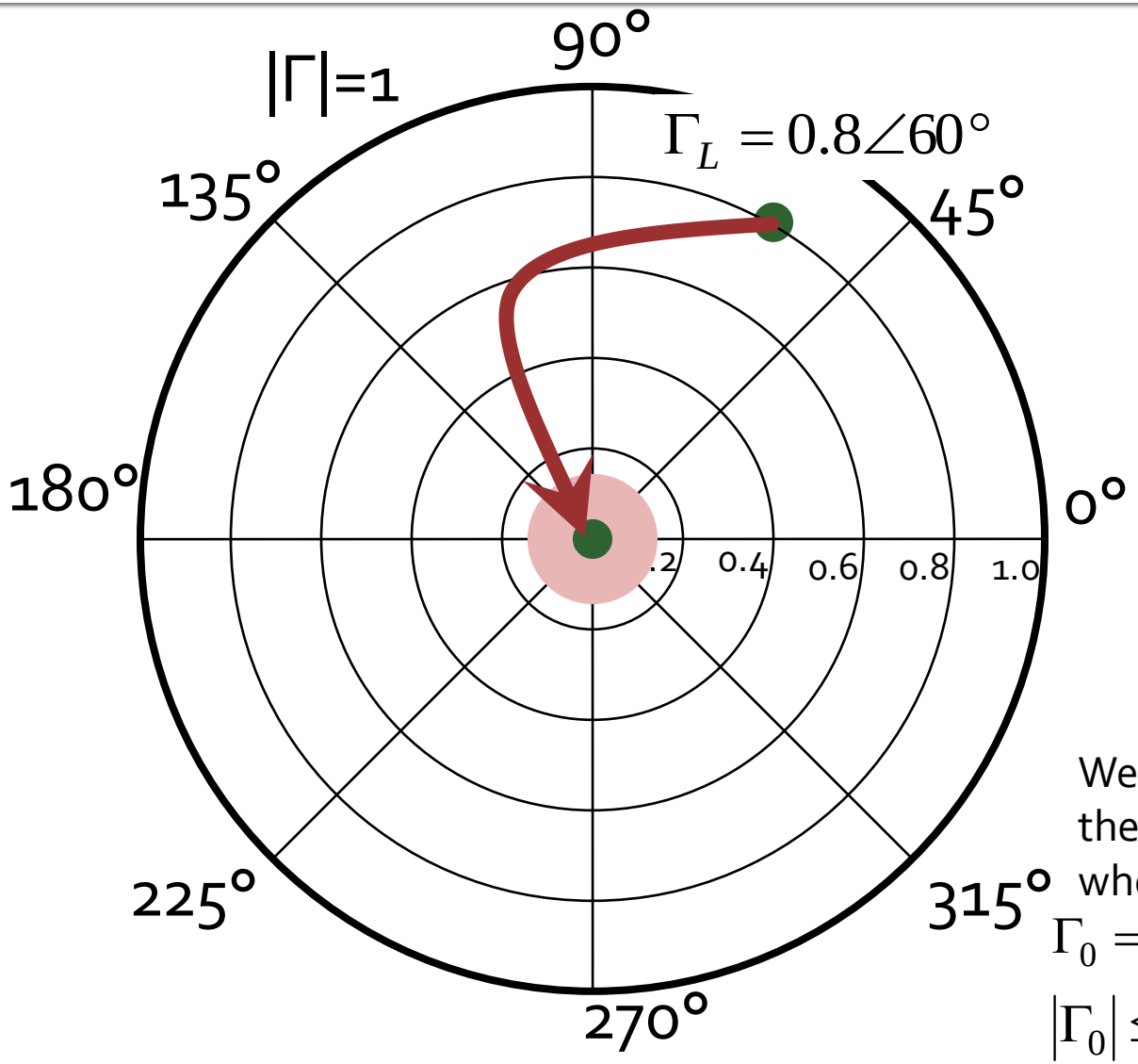
$$\Gamma_{in} = \Gamma_S^* \qquad \Gamma_{out} = \Gamma_L^*$$

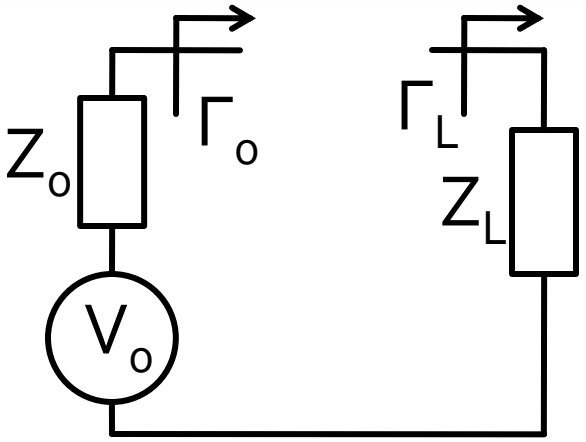
- For lossless matching sections

$$G_{T\max} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)\cdot\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S \cdot \Gamma_{in}|^2 \cdot |1-S_{22}\cdot\Gamma_L|^2} \qquad G_{T\max} = \frac{1}{1-|\Gamma_S|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

- For the general case of the bilateral transistor ($S_{12}\neq 0$) Γ_{in} and Γ_{out} depend on each other so the input and output sections must be matched simultaneously

The Smith Chart, matching, $Z_L \neq Z_0$





Matching Z_L load to Z_0 source.
We normalize Z_L over Z_0

$$Z_L = 21.429\,\Omega + j \cdot 82.479\,\Omega$$

$$z_L = 0.429 + j \cdot 1.65$$

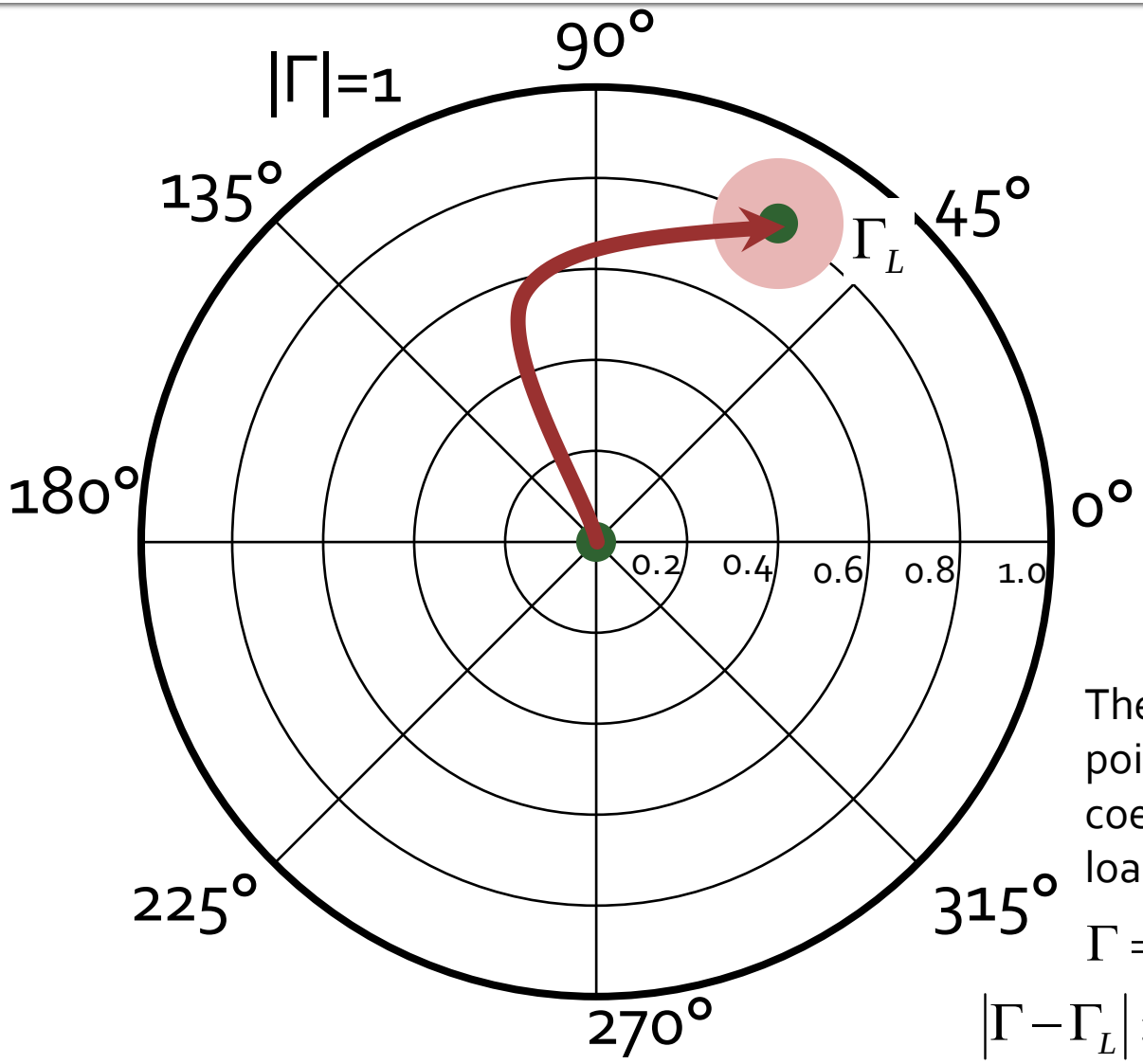
$$\Gamma_L = 0.8\angle 60°$$

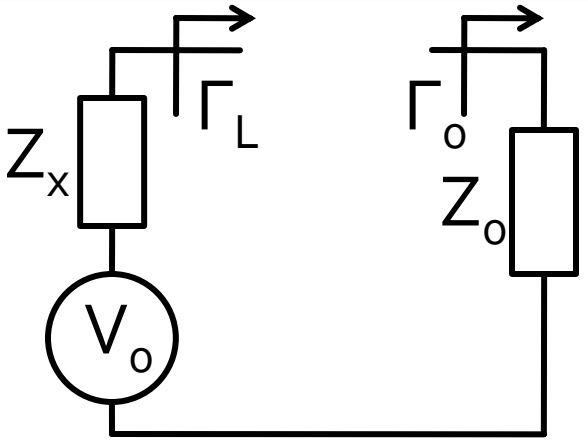
We must move the point denoting the reflection coefficient in the area where with a Z_0 source we have:

$\Gamma_0 = 0$ perfect match

$\left|\Gamma_0\right| \leq \Gamma_m$ "good enough" match

The Smith Chart, matching, $Z_L=Z_o$





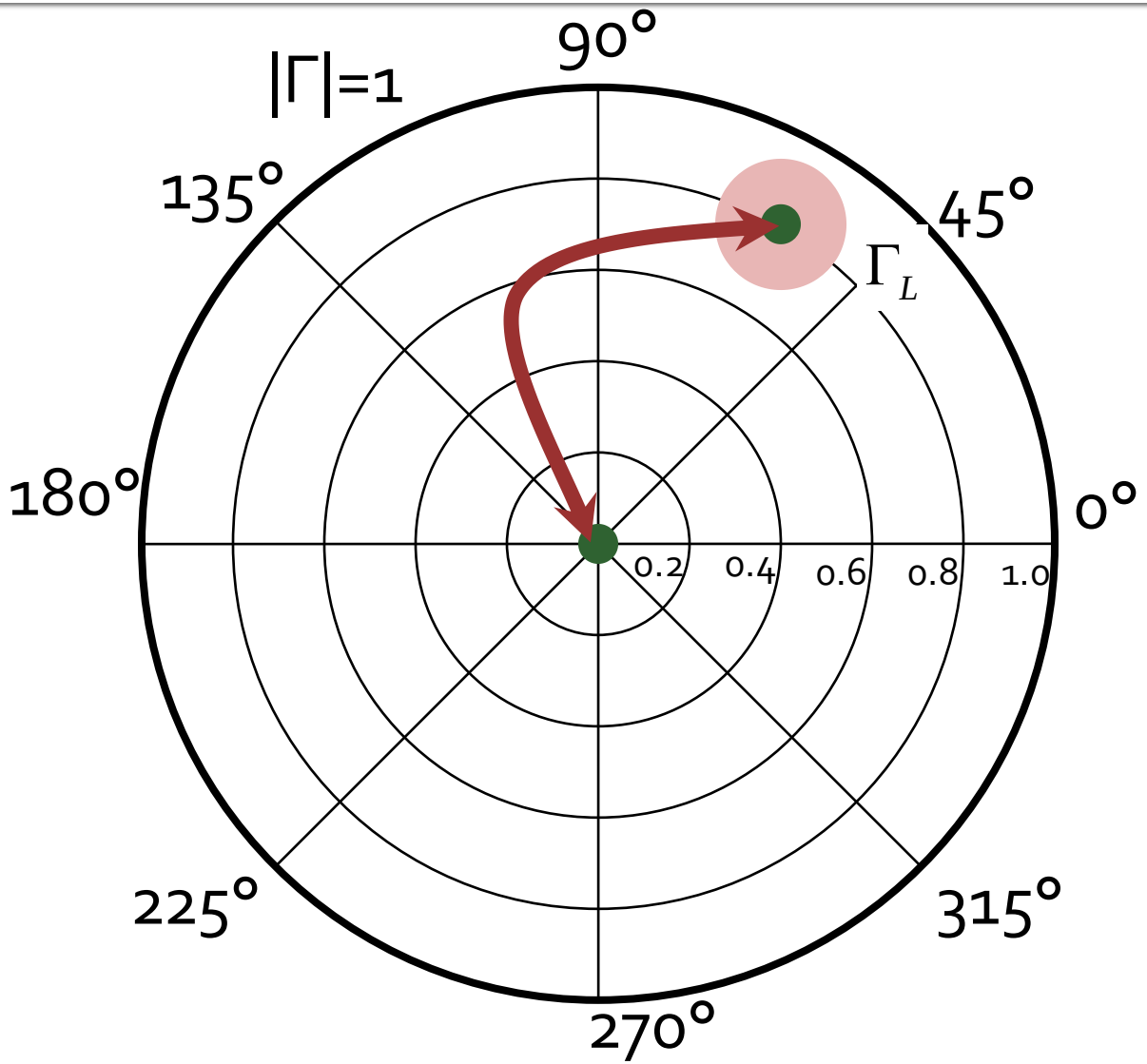
The source (eg. the transistor) having Z_X needs to see a certain reflection coefficient Γ_L towards the load Z_o

The matching circuit must move the point denoting the reflection coefficient in the area where for a Z_o load (Γ_o=o) we see towards it:

$\Gamma = \Gamma_L$ perfect match

$\left|\Gamma - \Gamma_L\right| \le \Gamma_m$ "good enough" match

The Smith Chart, matching , $Z_L \neq Z_0$, $Z_L = Z_0$



- The matching sections needed to move
  - Γ_L in Γ_0
  - Γ_0 in Γ_L
- are **identical**. They differ only by the **order** in which the elements are introduced into the matching circuit
- As a result, we can use in match design the same:
  - **methods**
  - **formulae**

Simultaneous matching

$$\Gamma_{in} = \Gamma_S^*$$

$$\Gamma_{in} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_S^* = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_L}{1 - S_{22} \cdot \Gamma_L}$$

$$\Gamma_{out} = \Gamma_L^*$$

$$\Gamma_{out} = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

$$\Gamma_L^* = S_{22} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

- We find Γ_S

$$\Gamma_S = S_{11}^* + \frac{S_{12}^* \cdot S_{21}^*}{1/\Gamma_L^* - S_{22}^*}$$

$$\Gamma_L^* = \frac{S_{22} - \Delta \cdot \Gamma_S}{1 - S_{11} \cdot \Gamma_S}$$

$$\Gamma_S \cdot \left(1 - |S_{22}|^2\right) + \Gamma_S^2 \cdot \left(\Delta \cdot S_{22}^* - S_{11}\right) = \Gamma_S \cdot \left(\Delta \cdot S_{11}^* \cdot S_{22}^* - |S_{22}|^2 - \Delta \cdot S_{12}^* \cdot S_{21}^*\right) +$$

$$+ S_{11}^* \cdot \left(1 - |S_{22}|^2\right) + S_{12}^* \cdot S_{21}^* \cdot S_{22}$$

Simultaneous matching

$$\Delta \cdot \left(S_{11}^* \cdot S_{22}^* - S_{12}^* \cdot S_{21}^* \right) = |\Delta|^2$$

$$\Gamma_S^2 \cdot \left(S_{11} - \Delta \cdot S_{22}^* \right) + \Gamma_S \cdot \left(|\Delta|^2 - |S_{11}|^2 + |S_{22}|^2 - 1 \right) + \left(S_{11}^* - \Delta^* \cdot S_{22} \right) = 0$$

- A quadratic equation

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1}$$

- Similarly

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C_2}$$

- With variables defined as:

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta \cdot S_{22}^* \end{cases} \quad \begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta \cdot S_{11}^* \end{cases}$$

Simultaneous matching

- Simultaneous matching is possible if:

$$B_1^2 - 4 \cdot |C_1|^2 > 0 \qquad B_2^2 - 4 \cdot |C_2|^2 > 0$$

$$\Delta \cdot \left(S_{11}^* \cdot S_{22}^* - S_{12}^* \cdot S_{21}^* \right) = |\Delta|^2$$

$$|C_1|^2 = \left| S_{11} - \Delta \cdot S_{22}^* \right|^2 = |S_{12}|^2 \cdot |S_{21}|^2 + \left(1 - |S_{22}|^2 \right) \cdot \left(|S_{11}|^2 - |\Delta|^2 \right)$$

$$B_1^2 - 4 \cdot |C_1|^2 = \left(1 + |S_{11}|^2 \right)^2 + \left(|S_{22}|^2 + |\Delta|^2 \right)^2 -$$
$$- 2 \cdot \left(1 + |S_{11}|^2 \right) \cdot \left(|S_{22}|^2 + |\Delta|^2 \right) - 4 \cdot |S_{12} \cdot S_{21}|^2 - 4 \cdot \left(1 - |S_{22}|^2 \right) \cdot \left(|S_{22}|^2 - |\Delta|^2 \right)$$

$$B_1^2 - 4 \cdot |C_1|^2 = \left(1 + |S_{11}|^2 \right)^2 + \left(|S_{22}|^2 + |\Delta|^2 \right)^2 -$$
$$- 4 \cdot |S_{11}|^2 - 4 \cdot |S_{22}|^2 \cdot |\Delta|^2 - 2 \cdot \left(1 - |S_{11}|^2 \right) \cdot \left(|S_{22}|^2 - |\Delta|^2 \right) - 4 \cdot |S_{12} \cdot S_{21}|^2$$

Simultaneous matching

$$B_1^2 - 4\cdot|C_1|^2 = \left(1+|S_{11}|^2\right)^2 + \left(|S_{22}|^2 + |\Delta|^2\right)^2 - 4\cdot|S_{11}|^2 - 4\cdot|S_{22}|^2\cdot|\Delta|^2 - 2\cdot\left(1-|S_{11}|^2\right)\cdot\left(|S_{22}|^2 - |\Delta|^2\right) - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(1-|S_{11}|^2\right)^2 + \left(|S_{22}|^2 - |\Delta|^2\right)^2 - 2\cdot\left(1-|S_{11}|^2\right)\cdot\left(|S_{22}|^2 - |\Delta|^2\right) - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2\right)^2 - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = \left(K\cdot 2\cdot|S_{12}\cdot S_{21}|\right)^2 - 4\cdot|S_{12}\cdot S_{21}|^2$$

$$B_1^2 - 4\cdot|C_1|^2 = 4\cdot|S_{12}|^2\cdot|S_{21}|^2\cdot\left(K^2-1\right)$$

- Similarly

$$B_2^2 - 4\cdot|C_2|^2 = 4\cdot|S_{12}|^2\cdot|S_{21}|^2\cdot\left(K^2-1\right)$$

Simultaneous matching

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1} \qquad \Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C_2}$$

- Solutions must ensure stability:

$$|\Gamma_S| < 1 \qquad |\Gamma_L| < 1$$

$$|\Delta| = |S_{11} \cdot S_{22} - S_{12} \cdot S_{21}| < 1$$

$$\begin{cases} B_1 > 0 \\ B_2 > 0 \end{cases}$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2 \cdot |S_{12} \cdot S_{21}|} > 1$$

$$\begin{cases} B_1^2 - 4 \cdot |C_1|^2 = 4 \cdot |S_{12}|^2 \cdot |S_{21}|^2 \cdot \left(K^2 - 1\right) > 0 \\ B_2^2 - 4 \cdot |C_2|^2 = 4 \cdot |S_{12}|^2 \cdot |S_{21}|^2 \cdot \left(K^2 - 1\right) > 0 \end{cases}$$

Simultaneous matching

- Simultaneous matching can be achieved **if and only if** the amplifier is **unconditionally stable** at the operating frequency, and $|\Gamma|<1$ solutions are those with "–" sign of quadratic solutions

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4\cdot|C_1|^2}}{2\cdot C_1}$$

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta\cdot S_{22}^* \end{cases}$$

$$\Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4\cdot|C_2|^2}}{2\cdot C_2}$$

$$\begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta\cdot S_{11}^* \end{cases}$$

Simultaneous matching

- In the case of the simultaneous matching the amplifier achieves the maximum transducer power gain for the bilateral transistor

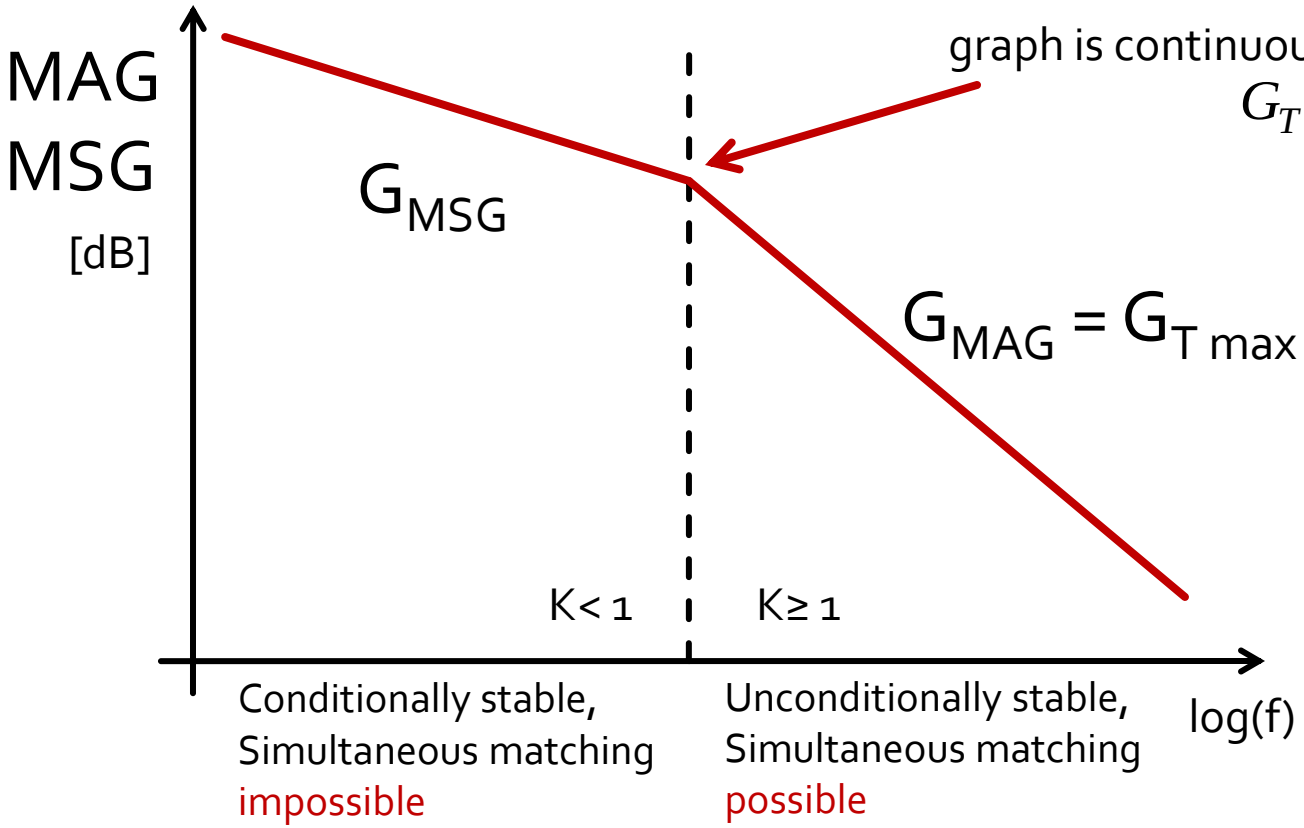
$$G_{T\max} = \frac{|S_{21}|}{|S_{12}|} \cdot \left(K - \sqrt{K^2 - 1}\right)$$

- If the device is **not unconditionaly stable** at a certain frequency we can use MSG (Maximum Stable Gain) as an indicator of the capability to obtain a power gain in stable conditions

$$G_{MSG} = \frac{|S_{21}|}{|S_{12}|}$$

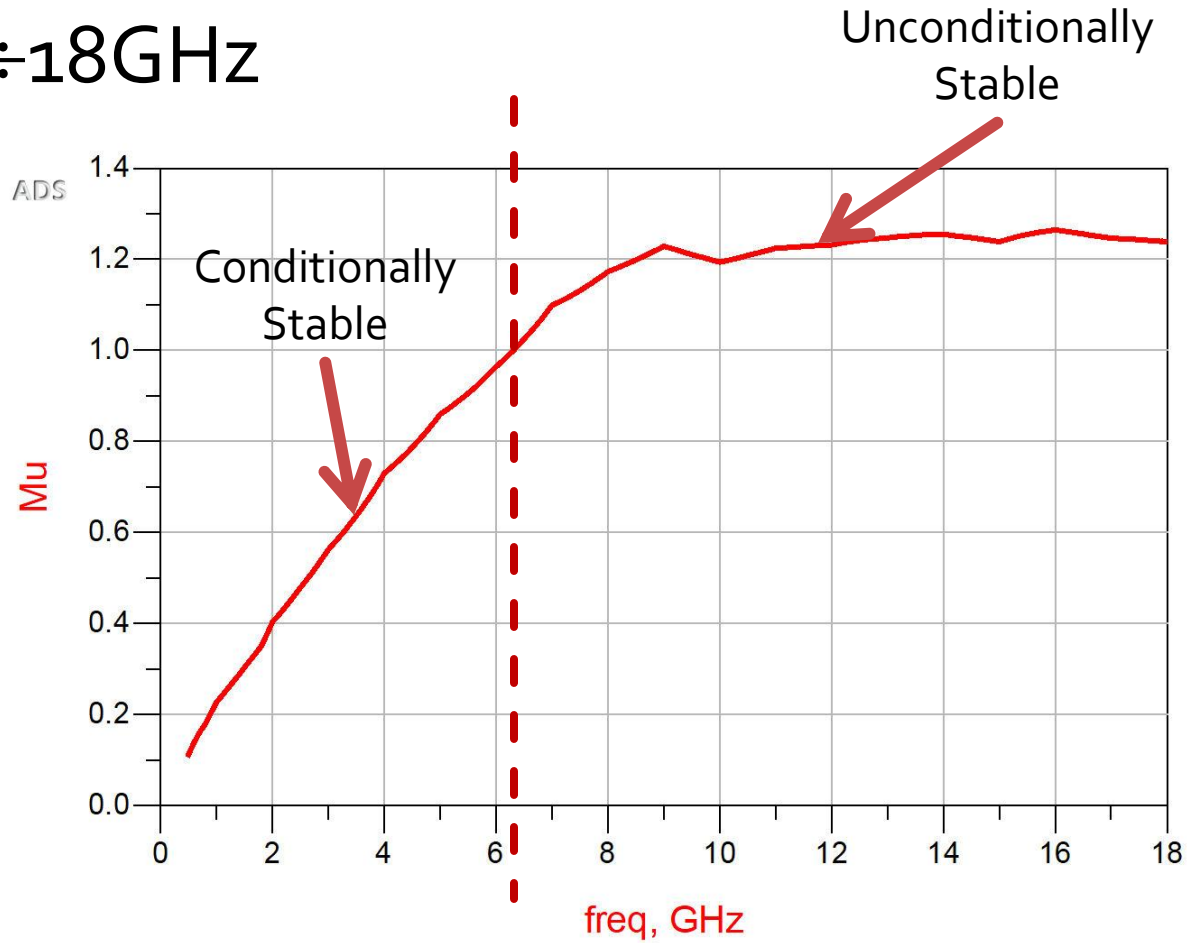
Maximum Available Gain

- Indicator across full frequency range of the capability to obtain a power gain



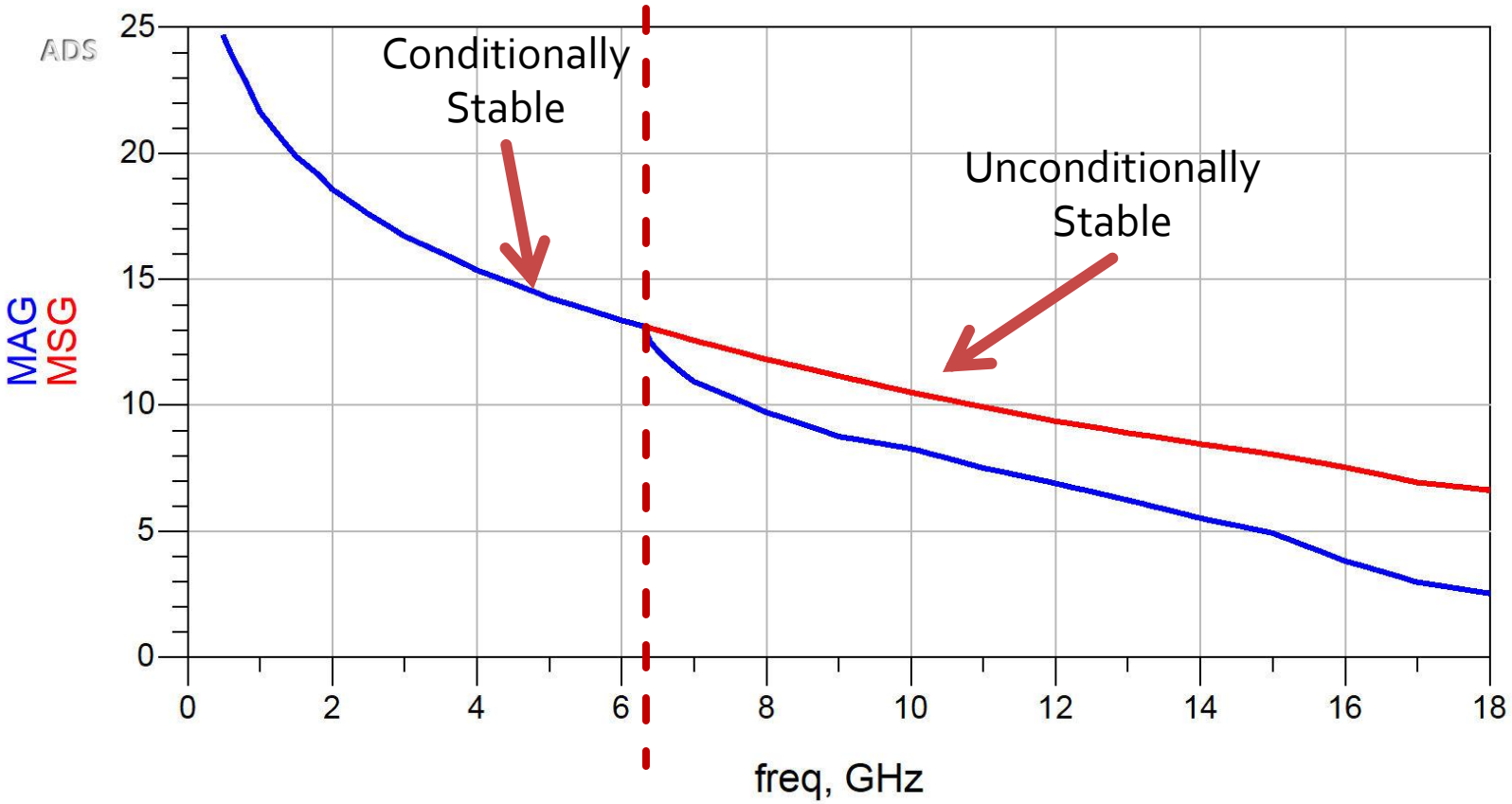
Stability

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



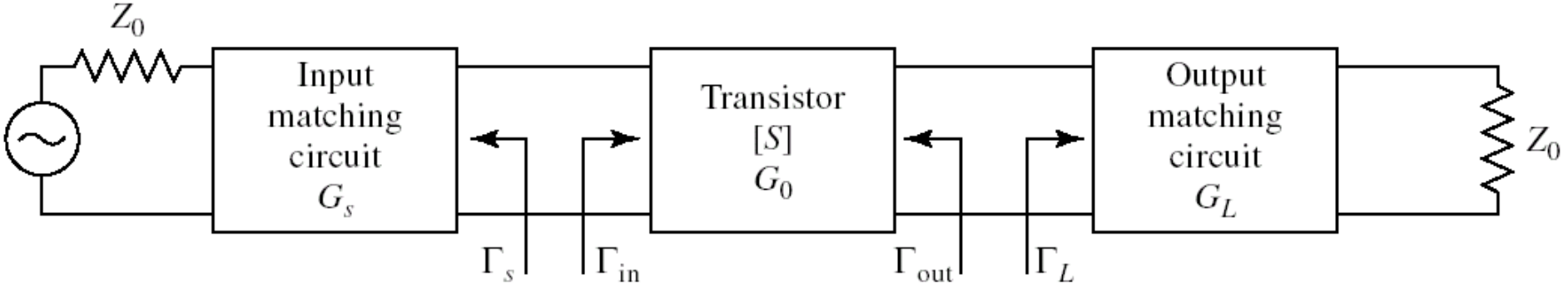
MAG/MSG

- ATF-34143 **at Vds=3V Id=20mA**.
- @0.5÷18GHz



Simultaneous matching, unilateral transistor

- In the case of **unilateral** amplifier/transistor (S12 = 0) simultaneous matching implies:



$$\Gamma_{in} = S_{11} \qquad \Gamma_{out} = S_{22}$$

$$\Gamma_S = S_{11}^* \qquad \Gamma_L = S_{22}^*$$

$$G_{T\max} = \frac{1}{1-|\Gamma_S|^2} \cdot |S_{21}|^2 \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2} \qquad G_{TU\max} = \frac{1}{1-|S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-|S_{22}|^2}$$

Example

- ATF-34143 **at Vds=3V Id=20mA**.
  - without stabilization K = **0.886**, MAG = 14.248dB @ 5GHz
  - cannot be used with this bias conditions
- ATF-34143 **at Vds=4V Id=40mA**
  - without stabilization K = **1.031**, MAG = 12.9dB @ 5GHz
  - we use this bias conditions for simultaneous matching

Example

- ATF-34143 **at Vds=4V Id=40mA**.
- @5GHz
  - S11 = 0.64∠111°
  - S12 = 0.117∠-27°
  - S21 = 2.923 ∠-6°
  - S22 = 0.21 ∠111°

Computations

- Complex S Parameters
  - S11 = -0.229+0.597·j
  - S12 = 0.104-0.053·j
  - S21 = 2.907-0.306·j
  - S22 = -0.075+0.196·j

$$\begin{cases} S_{11} = 0.64\angle 111^\circ \\ S_{11} = 0.64 \cdot \cos 111^\circ + j \cdot 0.64 \cdot \sin 111^\circ \end{cases}$$

$$G_{T\max} = \frac{|S_{21}|}{|S_{12}|} \cdot \left(K - \sqrt{K^2 - 1}\right) = 19.497 = 12.9\,\text{dB}$$

$$G_{TU\max} = \frac{1}{1 - |S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1 - |S_{22}|^2} = 15.139 = 11.8\,\text{dB}$$

Computations

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta \cdot S_{22}^* \end{cases} \qquad \begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta \cdot S_{11}^* \end{cases}$$

$$\begin{cases} B_1 = ? \\ C_1 = ? \end{cases} \qquad \begin{cases} B_2 = ? \\ C_2 = ? \end{cases}$$

$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1} \qquad \Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C_2}$$

$$\Gamma_S = ? \qquad \Gamma_L = ?$$

Computations

$$\begin{cases} B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ C_1 = S_{11} - \Delta \cdot S_{22}^* \end{cases} \qquad \begin{cases} B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ C_2 = S_{22} - \Delta \cdot S_{11}^* \end{cases}$$

$$\begin{cases} B_1 = 1.207 \\ C_1 = -0.277 + j \cdot 0.529 \end{cases} \qquad \begin{cases} B_2 = 0.476 \\ C_2 = -0.222 - j \cdot 0.013 \end{cases}$$

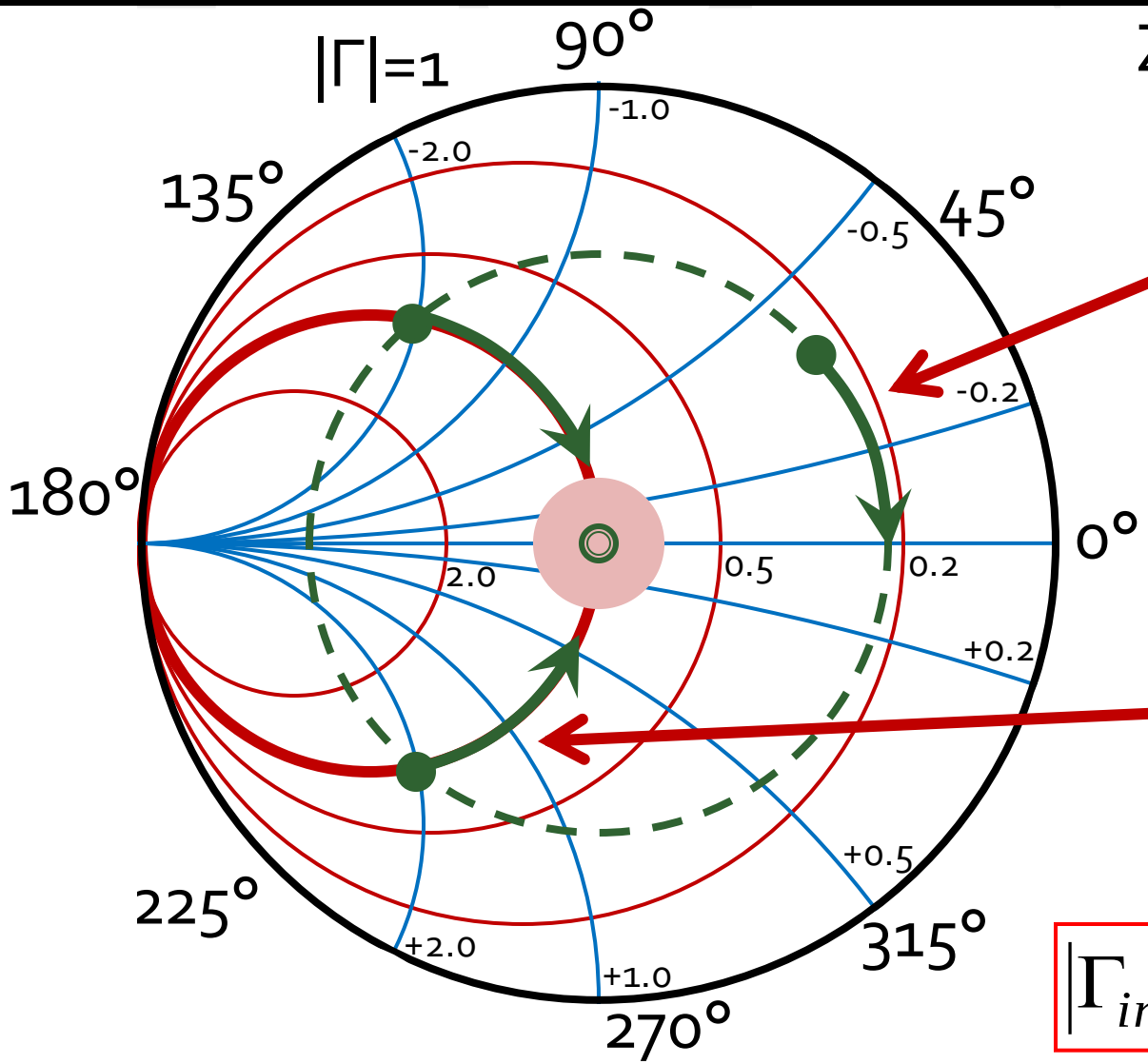
$$\Gamma_S = \frac{B_1 - \sqrt{B_1^2 - 4 \cdot |C_1|^2}}{2 \cdot C_1} \qquad \Gamma_L = \frac{B_2 - \sqrt{B_2^2 - 4 \cdot |C_2|^2}}{2 \cdot C_2}$$

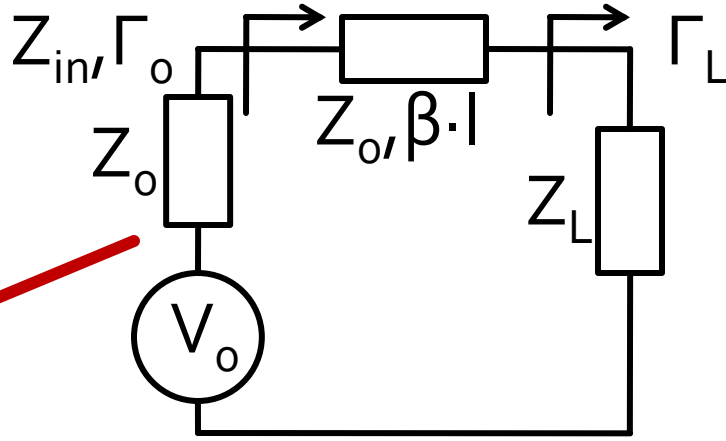
$$\Gamma_S = -0.403 - j \cdot 0.768 \qquad \Gamma_L = -0.685 + j \cdot 0.04$$

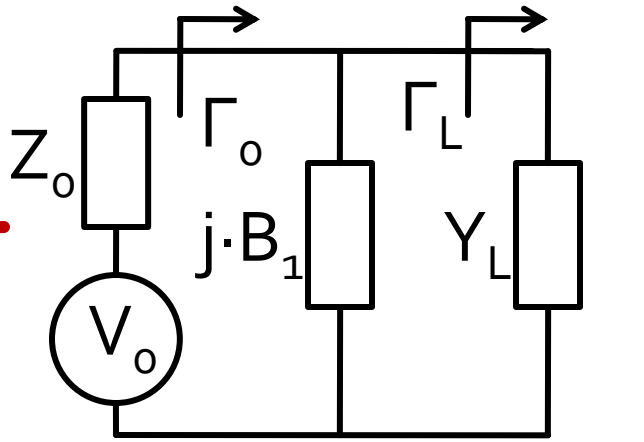
$$|\Gamma_S| = 0.867 < 1 \qquad |\Gamma_L| = 0.686 < 1$$

$$\Gamma_S = 0.867 \angle -117.7^\circ \qquad \Gamma_L = 0.686 \angle 176.7^\circ$$

Shunt stub matching, L7







$$|\Gamma_{in}| = |\Gamma_L|$$

$$g_{in} = 1$$

Analytical solution (Γ_S)

$$\cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.867 \angle -117.7°$$

$$|\Gamma_S| = 0.867; \quad \varphi = -117.7° \qquad \cos(\varphi + 2\theta) = -0.867 \Rightarrow \quad (\varphi + 2\theta) = \pm 150.1°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

$$(-117.7° + 2\theta) = +150.1° \qquad \theta = 133.9° \qquad \mathrm{Im}\, y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -3.477$$

$$\theta_{sp} = \tan^{-1}(\mathrm{Im}\, y_S) = -74°(+180°) \rightarrow \theta_{sp} = 106°$$

  - **"-" solution**

$$(-117.7° + 2\theta) = -150.1° \qquad \theta = -16.2°(+180°) \rightarrow \theta = 163.8°$$

$$\mathrm{Im}\, y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +3.477 \qquad \theta_{sp} = \tan^{-1}(\mathrm{Im}\, y_S) = 74°$$

Analytical solution (Γ_L)

$$\cos(\varphi + 2\theta) = -|\Gamma_L|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}}$$

$$\Gamma_L = 0.686 \angle 176.7°$$

$$|\Gamma_L| = 0.686; \quad \varphi = 176.7°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**
  - **"-" solution**

Analytical solution (Γ_L)

$$\cos(\varphi + 2\theta) = -|\Gamma_L|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}}$$

$$\Gamma_L = 0.686 \angle 176.7°$$

$$|\Gamma_L| = 0.686; \quad \varphi = 176.7° \qquad \cos(\varphi + 2\theta) = -0.686 \Rightarrow \quad (\varphi + 2\theta) = \pm 133.3°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

$$(176.7° + 2\theta) = +133.3° \qquad \theta = -21.7°(+180°) \rightarrow \theta = 158.3°$$

$$\theta_{sp} = \tan^{-1}(\mathrm{Im}\, y_L) = -62.1°(+180°) \rightarrow \theta_{sp} = 117.9° \qquad \mathrm{Im}\, y_L = \frac{-2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}} = -1.885$$

  - **"-" solution**

$$(176.7° + 2\theta) = -133.3° \qquad \theta = -155°(+180°) \rightarrow \theta = 25°$$

$$\mathrm{Im}\, y_L = \frac{+2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}} = +1.885 \qquad \theta_{sp} = \tan^{-1}(\mathrm{Im}\, y_L) = 62.1°$$

Complete analytical solution

- We choose **one** of the two possible solutions for the input matching

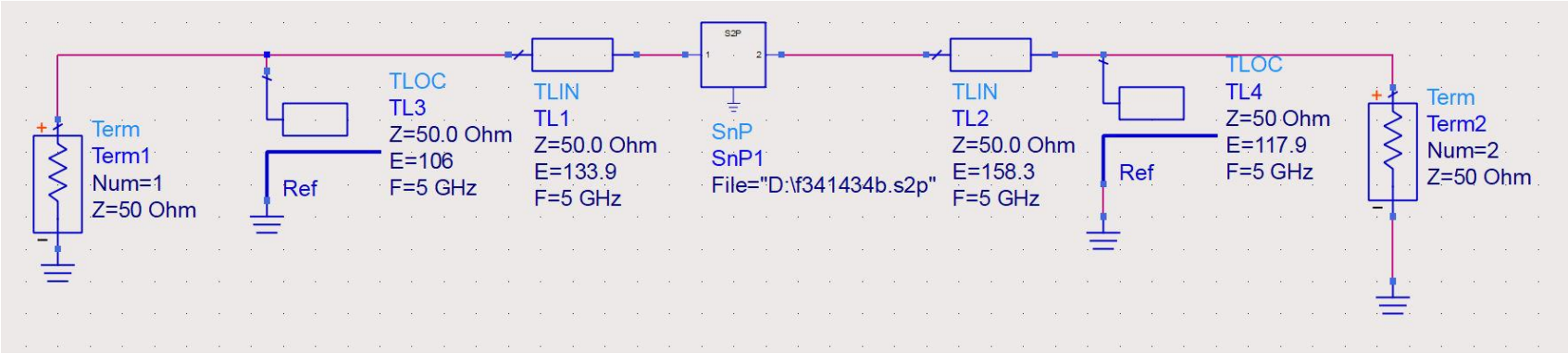
$$(\varphi+2\theta)=\begin{cases}+150.1^\circ\\-150.1^\circ\end{cases}\quad\theta=\begin{cases}133.9^\circ\\163.8^\circ\end{cases}\quad \mathrm{Im}[y_S(\theta)]=\begin{cases}-3.477\\+3.477\end{cases}\quad\theta_{sp}=\begin{cases}-74^\circ+180^\circ=106^\circ\\+74^\circ\end{cases}$$

- Similarly for the output matching

$$(\varphi+2\theta)=\begin{cases}+133.3^\circ\\-133.3^\circ\end{cases}\quad\theta=\begin{cases}158.3^\circ\\25.0^\circ\end{cases}\quad \mathrm{Im}[y_S(\theta)]=\begin{cases}-1.885\\+1.885\end{cases}\quad\theta_{sp}=\begin{cases}117.9^\circ\\62.1^\circ\end{cases}$$

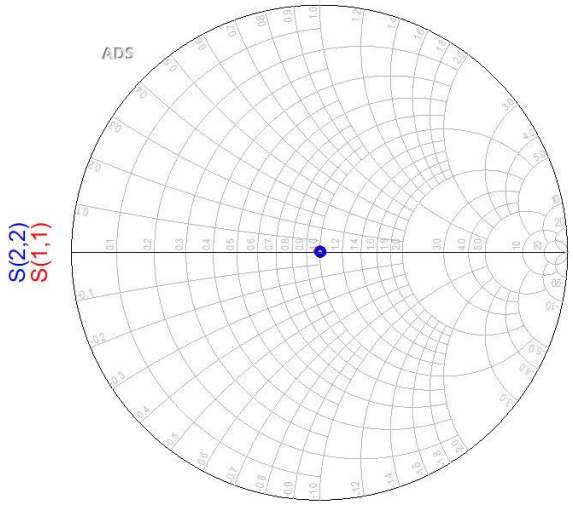
- In total there are **4** possible solutions input/output

ADS



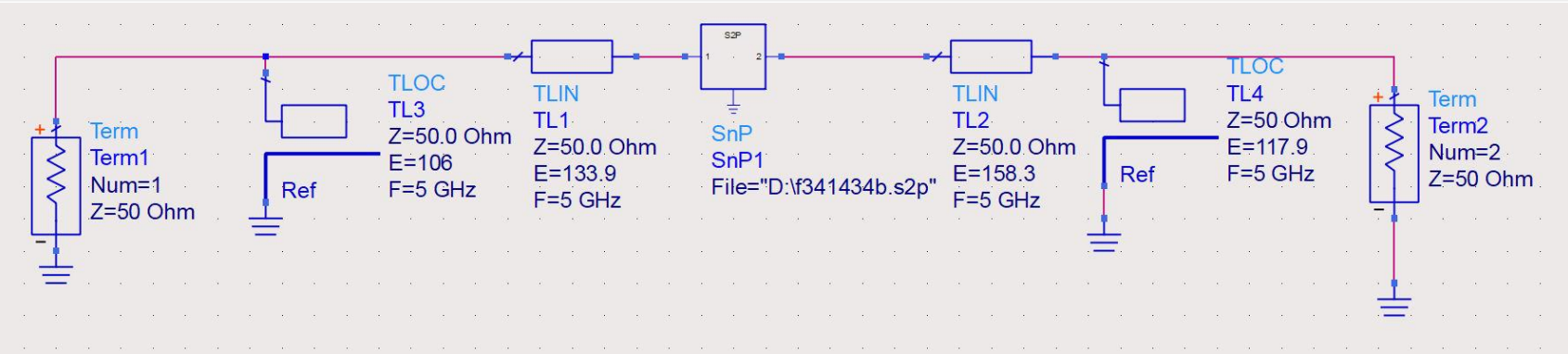
Eqn GT=10*log(mag(S(2,1))**2)

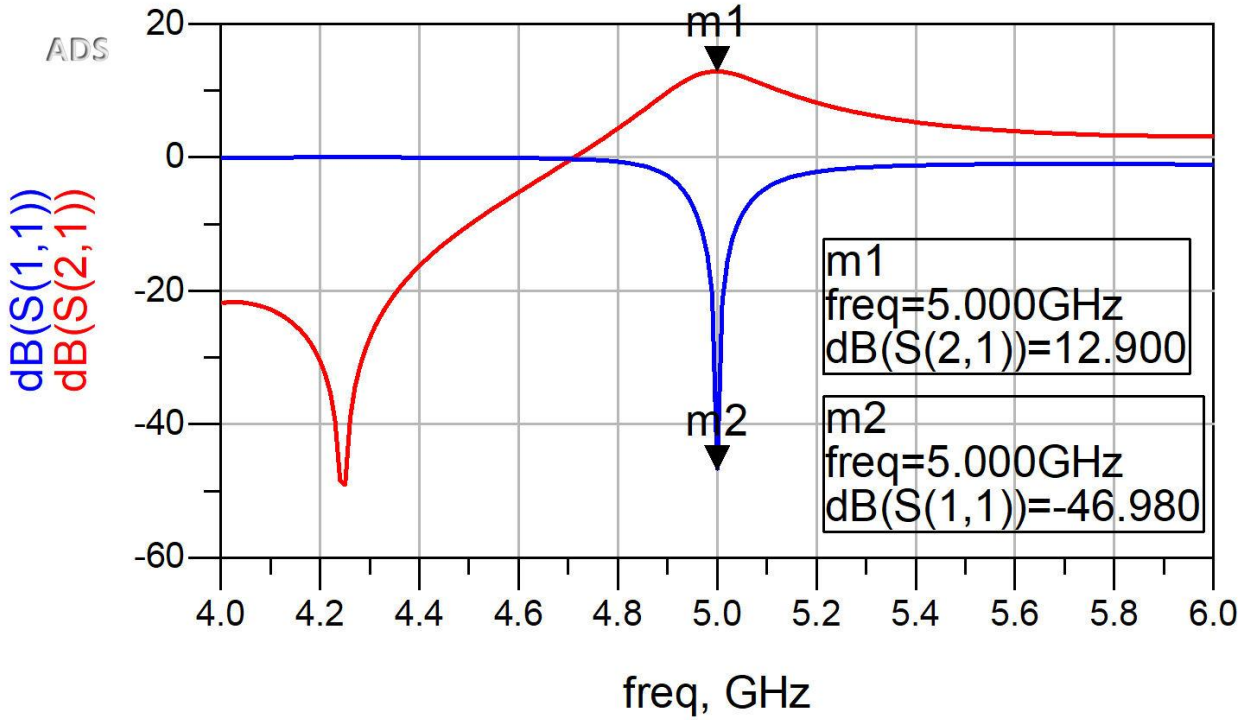
freq	S(2,1)	GT	S(1,1)	S(2,2)
5.000 GHz	4.415 / 157.353	12.900	0.004 / 86.088	0.004 / 37.766



freq (5.000GHz to 5.000GHz)

ADS

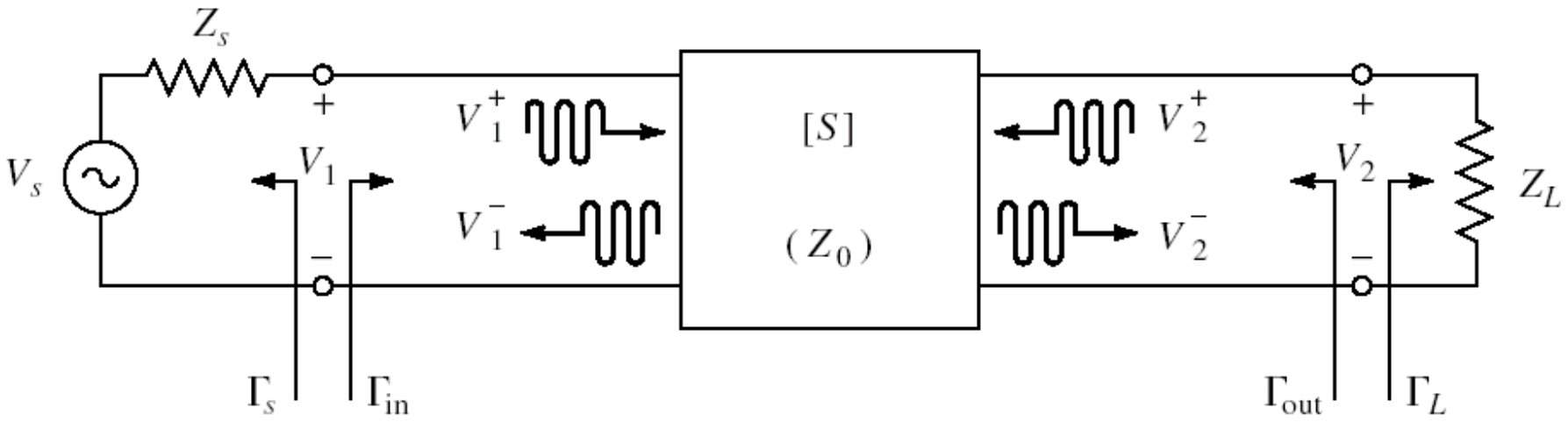

Term
Term1
Num=1
Z=50 Ohm
Ref
TLOC
TL3
Z=50.0 Ohm
E=106
F=5 GHz
TLIN
TL1
Z=50.0 Ohm
E=133.9
F=5 GHz
S2P
SnP
SnP1
File="D:\f341434b.s2p"
TLIN
TL2
Z=50.0 Ohm
E=158.3
F=5 GHz
Ref
TLOC
TL4
Z=50 Ohm
E=117.9
F=5 GHz
Term
Term2
Num=2
Z=50 Ohm


ADS
m1
m2
m1
freq=5.000GHz
dB(S(2,1))=12.900
m2
freq=5.000GHz
dB(S(1,1))=-46.980
dB(S(1,1))
dB(S(2,1))
20
0
-20
-40
-60
4.0
4.2
4.4
4.6
4.8
5.0
5.2
5.4
5.6
5.8
6.0
freq, GHz

Microwave Amplifiers

Design for Specified Gain

Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - **power gain**
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

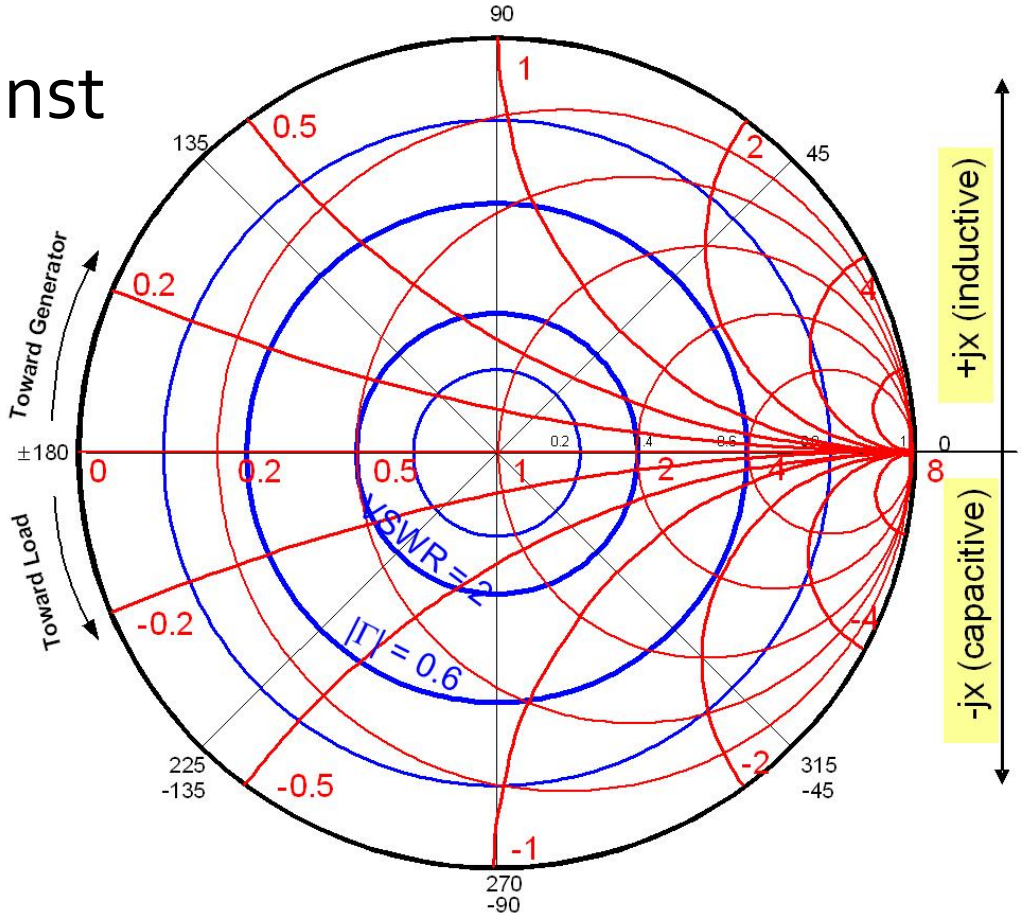
Design for Specified Gain

- In many cases we need an approach other than "brute force" when we prefer to design for **less than the maximum obtainable gain**, in order to:
  - improve noise behavior (Lab 3 + Lect. 8 next)
  - improve stability
  - improve VSWR
  - control performance at multiple frequencies
  - improve amplifier's bandwidth

Constant VSWR circles

- Certain applications may require a certain ratio between maximum / minimum line voltage
  - VSWR = const → Γ = const

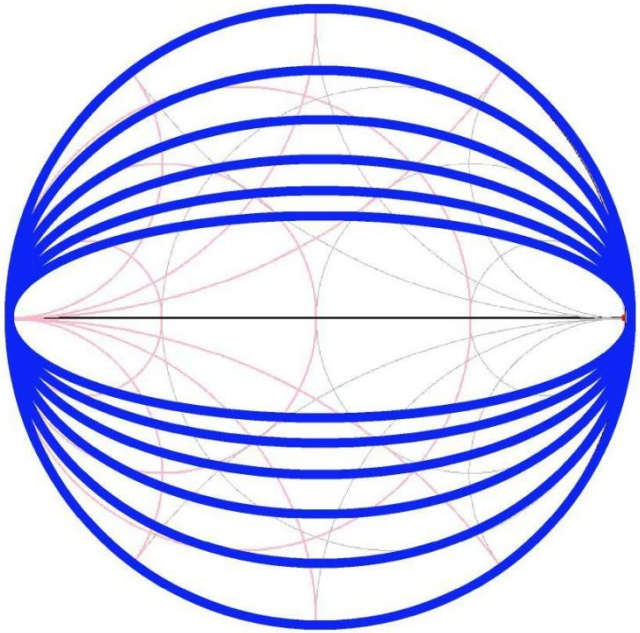
$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

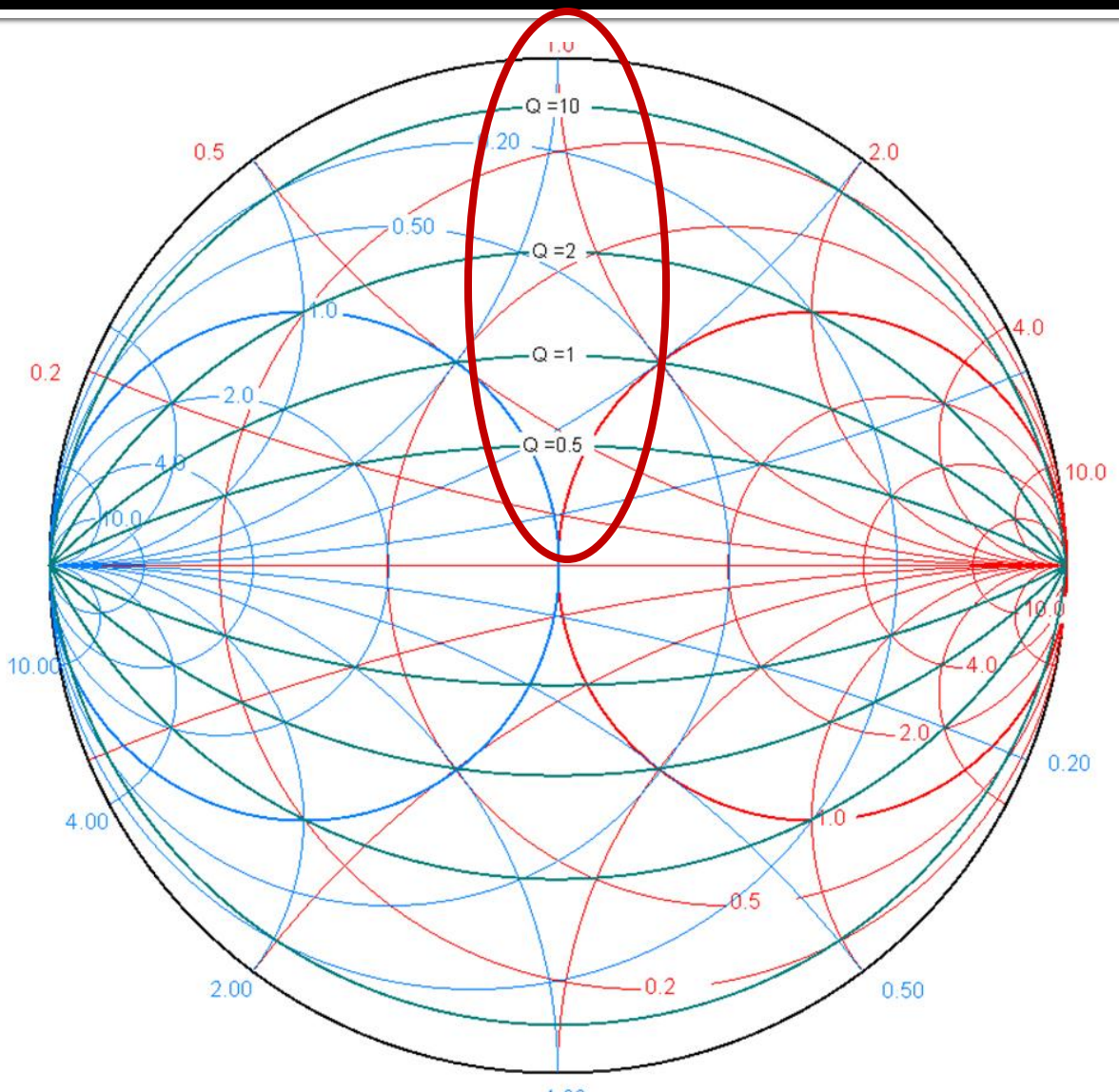


Constant Q circles

- Quality factor Q

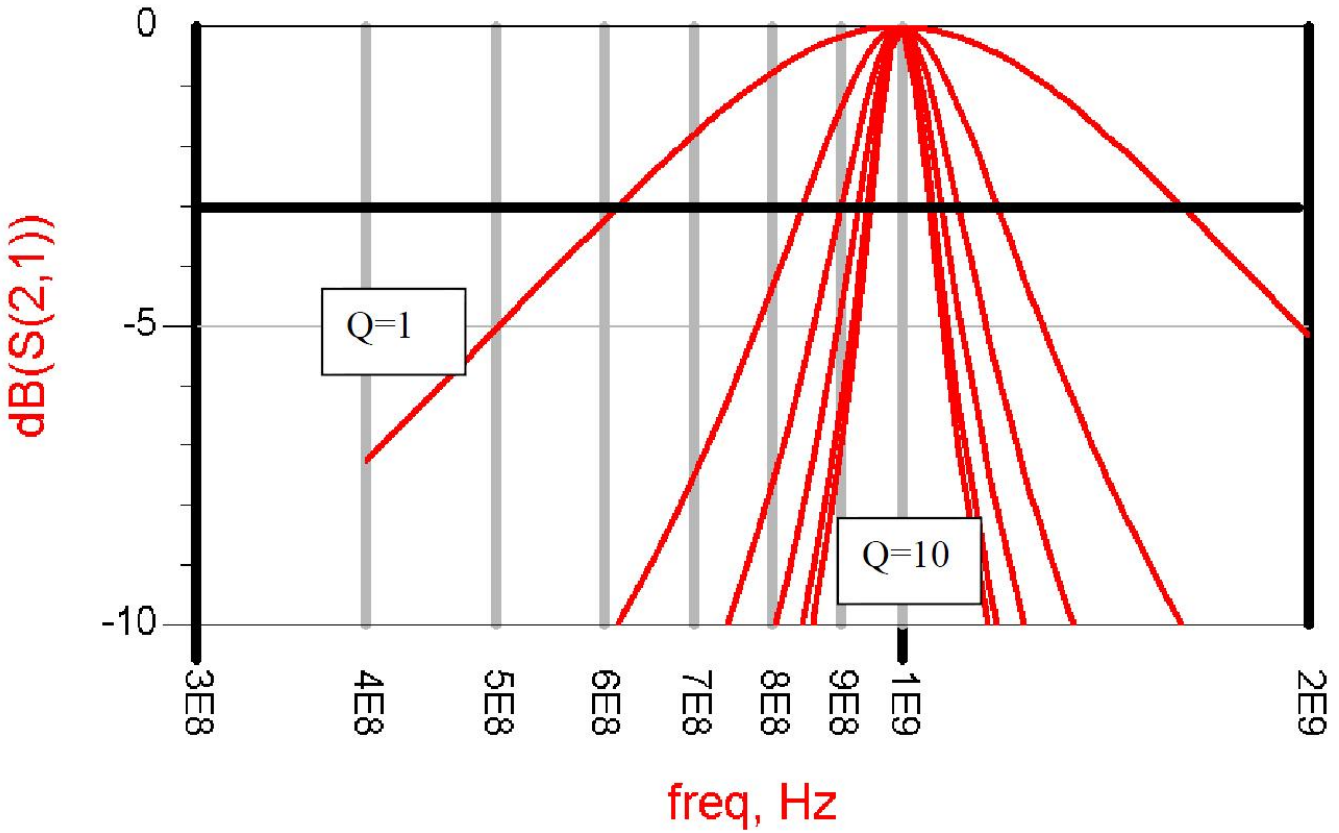
$$Q = \frac{X}{R} = \frac{G}{B} = const$$





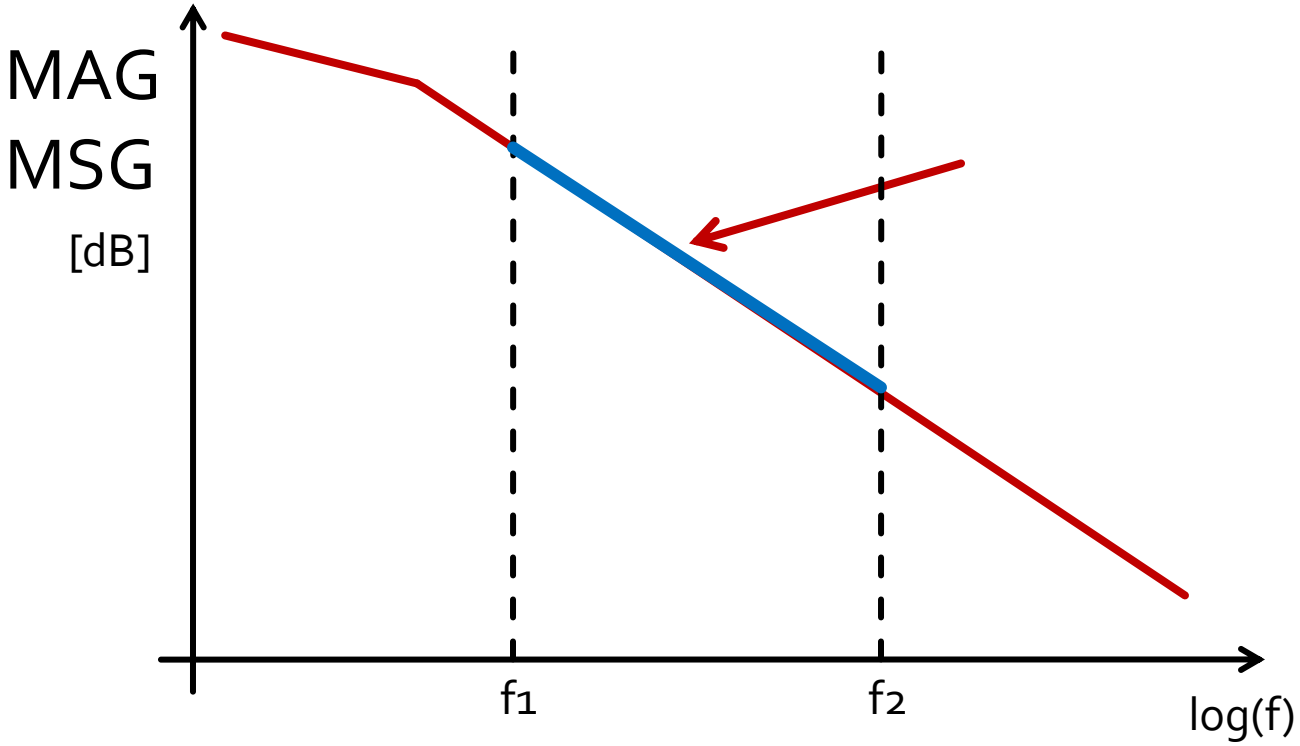
Quality factor - bandwidth

- High quality factor is equivalent with narrow bandwidth



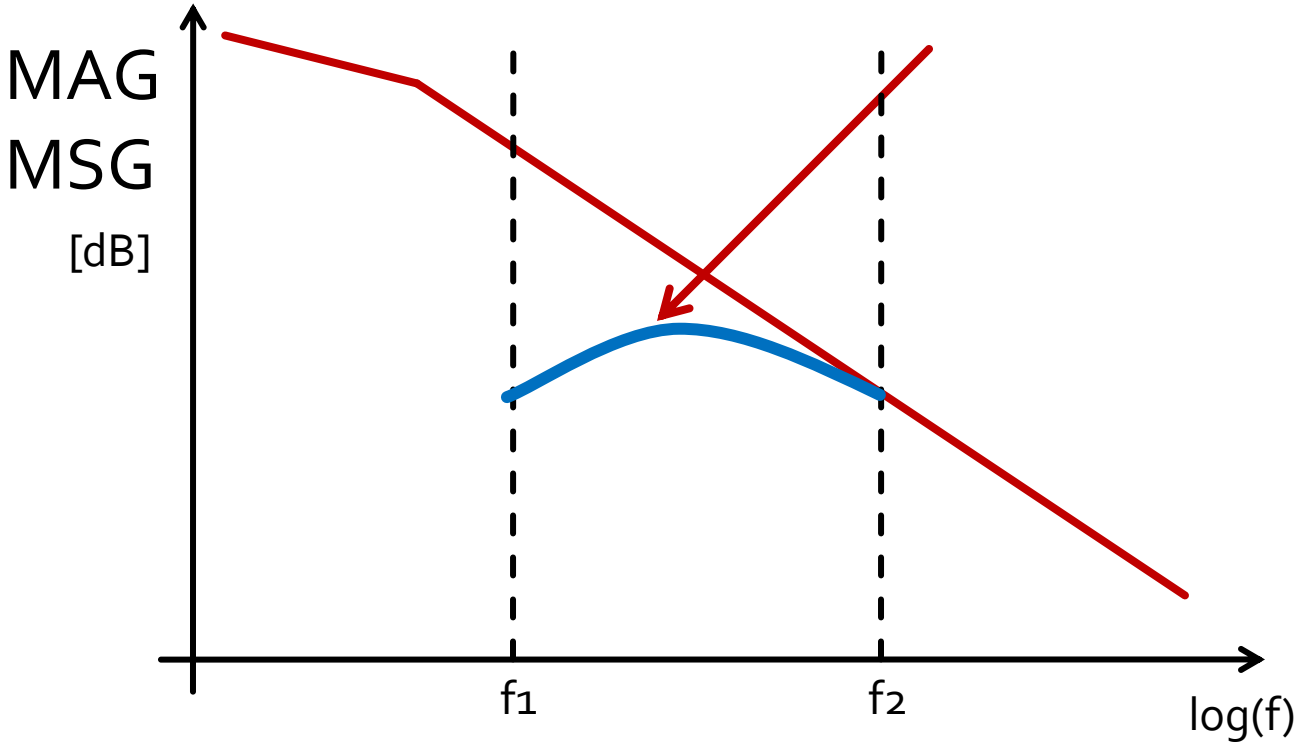
Wide bandwidth amplifier

- Design for maximum gain at two different frequencies creates an frequency unbalanced amplifier



Wide bandwidth amplifier

- Design for maximum gain at highest frequency
- Controlled mismatch at lower frequency
  - eventually at more frequencies inside the bandwidth



Design for Specified Gain

- Assumes the amplifier device **unilateral**

Input and output can be treated independently

$$G_{TU} = |S_{21}|^2 \cdot \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2} \qquad S_{12} \cong 0 \qquad \Gamma_{in} = S_{11}$$

- Maximum power gain

$$\Gamma_S = S_{11}^*$$

$$\Gamma_L = S_{22}^*$$

$$G_{TU\,\max} = \frac{1}{1-|S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-|S_{22}|^2}$$

Unilateral figure of merit

- Allows estimation of the error introduced by the unilateral assumption

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} \qquad U = \frac{|S_{12}| \cdot |S_{21}| \cdot |S_{11}| \cdot |S_{22}|}{\left(1-|S_{11}|^2\right) \cdot \left(1-|S_{22}|^2\right)}$$

- We compute U then the maximum and minimum deviation of G_{TU} from G_T
  - this deviation must be accounted in the design as a reserve gain against the target gain

$$-20 \cdot \log(1+U) < G_T[dB] - G_{TU}[dB] < -20 \cdot \log(1-U)$$

Example

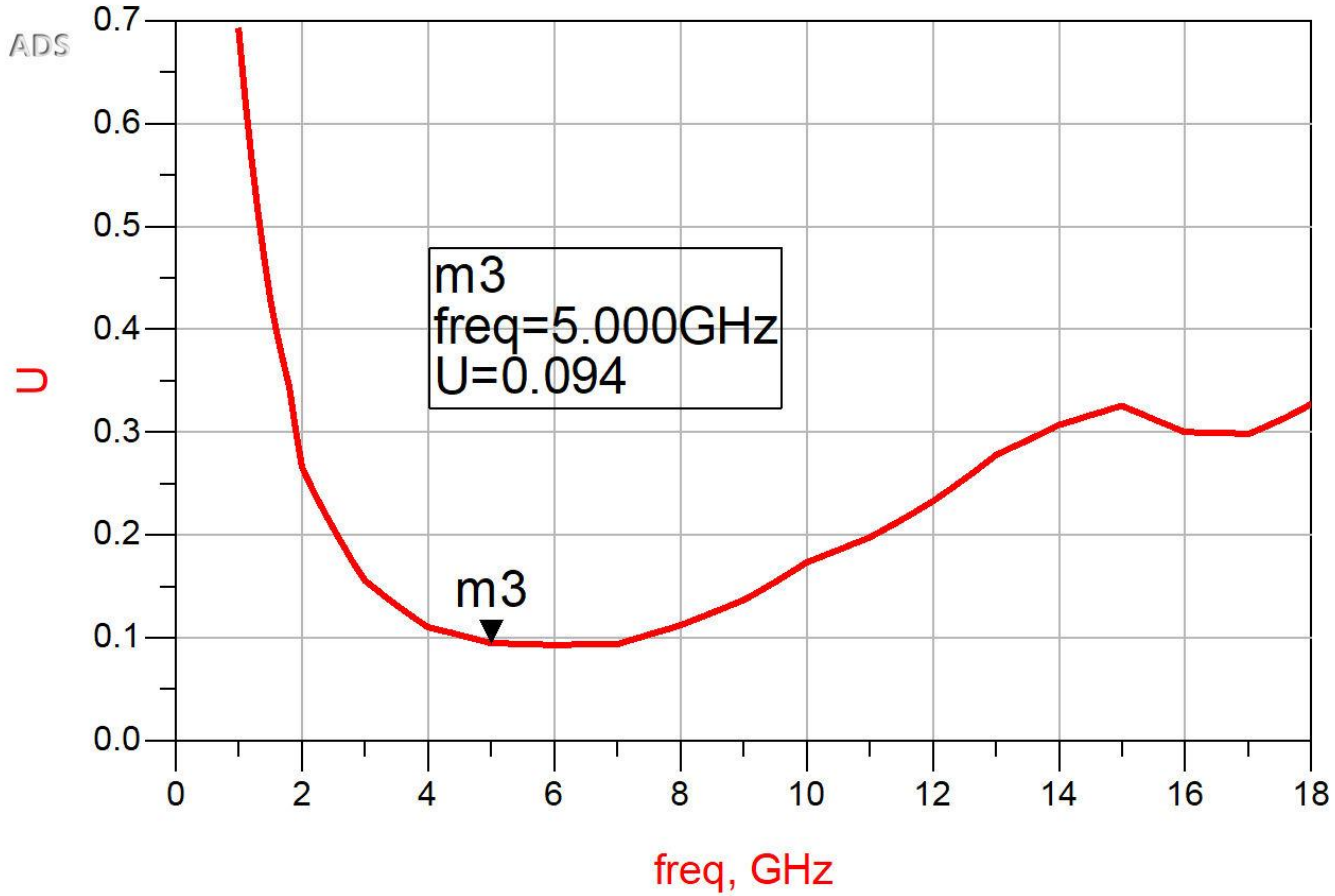
- ATF-34143 **at Vds=3V Id=20mA**.
- @5GHz
  - S11 = 0.64∠139°
  - S12 = 0.119∠-21°
  - S21 = 3.165 ∠16°
  - S22 = 0.22 ∠146°

$$U = \frac{|S_{12}| \cdot |S_{21}| \cdot |S_{11}| \cdot |S_{22}|}{\left(1 - |S_{11}|^2\right) \cdot \left(1 - |S_{22}|^2\right)} = 0.094$$

$$-0.783\ dB < G_T[dB] - G_{TU}[dB] < 0.861\ dB$$

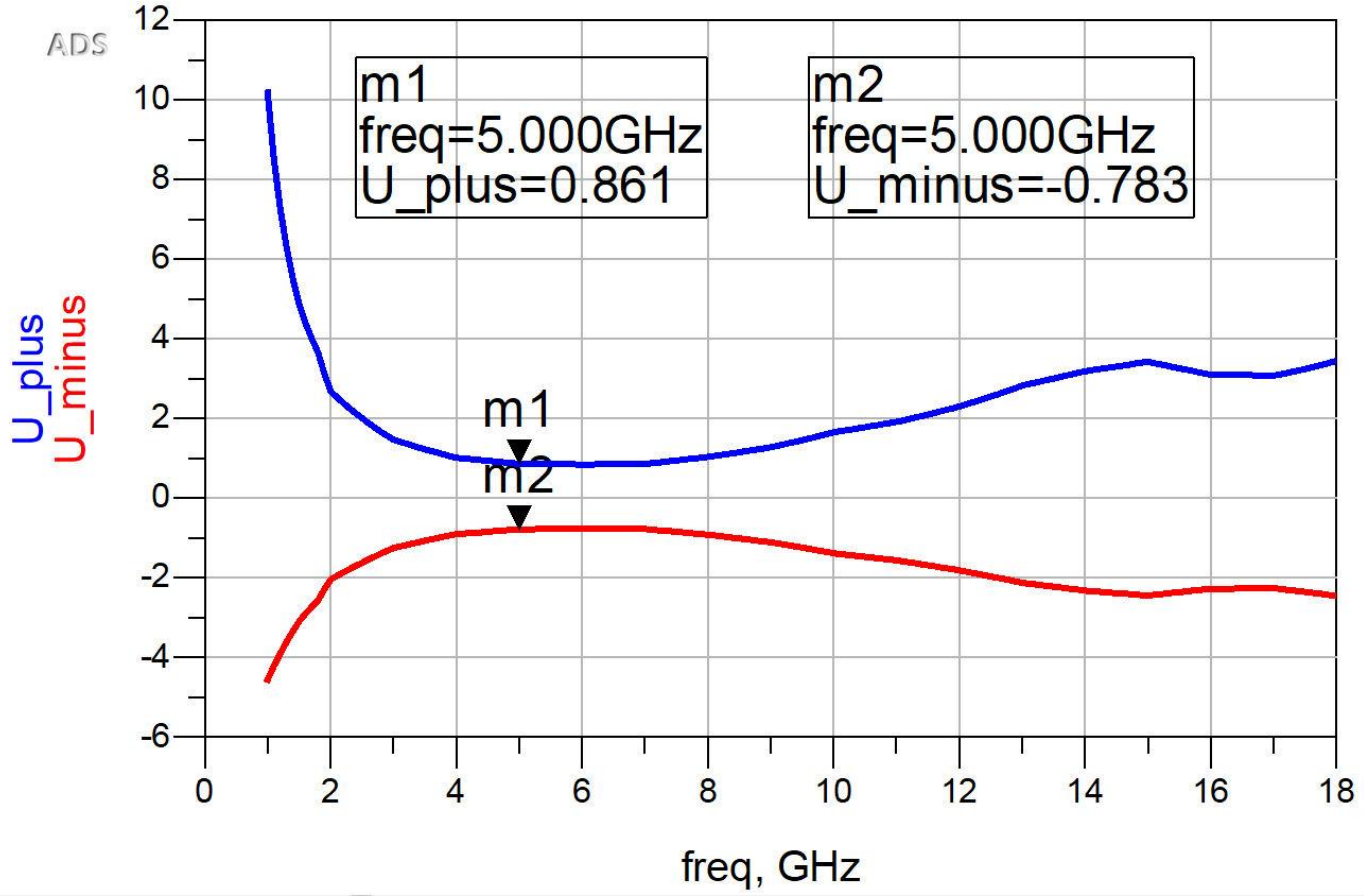
Example

- ATF-34143 **at Vds=3V Id=20mA**.
- @5GHz

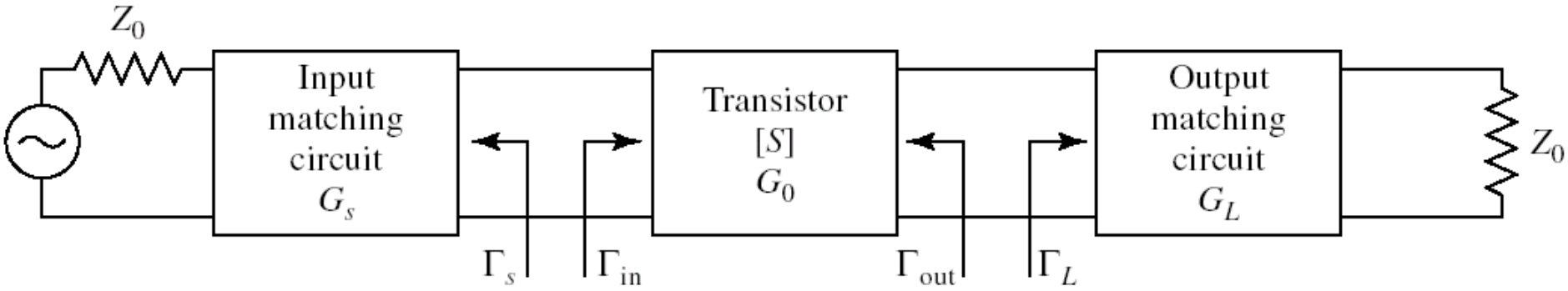


Example

- ATF-34143 **at Vds=3V Id=20mA**.
- @5GHz



Design for Specified Gain



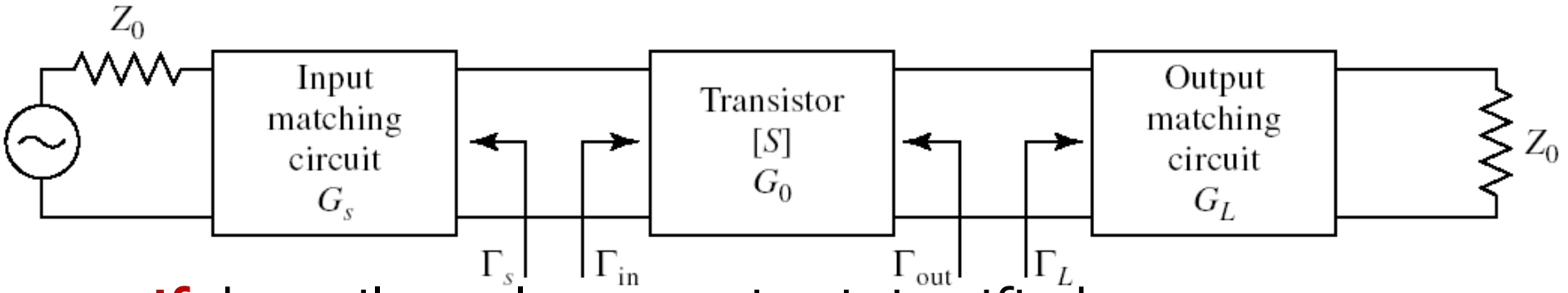
- In the unilateral assumption:

$$G_{TU} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\cdot|S_{21}|^2\cdot\frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \qquad G_0 = |S_{21}|^2 \qquad G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$G_S = G_S(\Gamma_S) \qquad G_L = G_L(\Gamma_L)$$

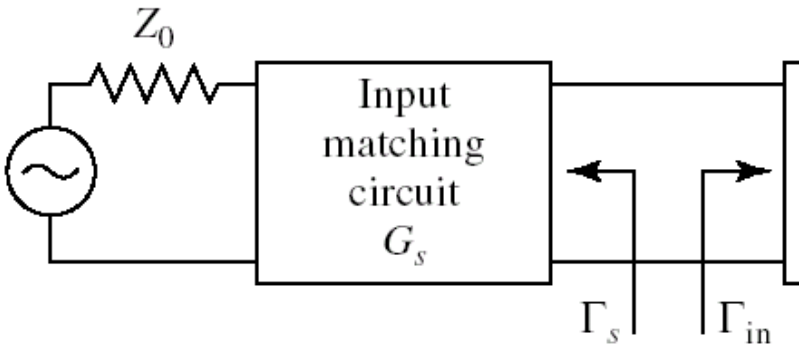
Design for Specified Gain



- **If** the unilateral assumption is justified :
  - power gain added by the input matching circuit **is not** influenced by the output matching circuit $G_S = G_S(\Gamma_S)$
  - power gain added by the output matching circuit **is not** influenced by the input matching circuit $G_L = G_L(\Gamma_L)$
- Output /Input match can be designed independently
  - We can impose different demands for input/output
  - Total gain is:

$$G_T = G_S \cdot G_0 \cdot G_L \qquad G_T[dB] = G_S[dB] + G_0[dB] + G_L[dB]$$

Input matching circuit



$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}$$

- Maximum gain in the case of complex conjugate match

$$\Gamma_S = S_{11}^* \Rightarrow \qquad G_{S\max} = \frac{1}{1-|S_{11}|^2}$$

- For any other input matching circuit:

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} < G_{S\max} = \frac{1}{1-|S_{11}|^2}$$

Example

- ATF-34143 **at Vds=3V Id=20mA**.
- @5GHz
  - S11 = 0.64∠139°
  - S12 = 0.119∠-21°
  - S21 = 3.165 ∠16°
  - S22 = 0.22 ∠146°

$$U = \frac{|S_{12}| \cdot |S_{21}| \cdot |S_{11}| \cdot |S_{22}|}{\left(1-|S_{11}|^2\right) \cdot \left(1-|S_{22}|^2\right)} = 0.094$$

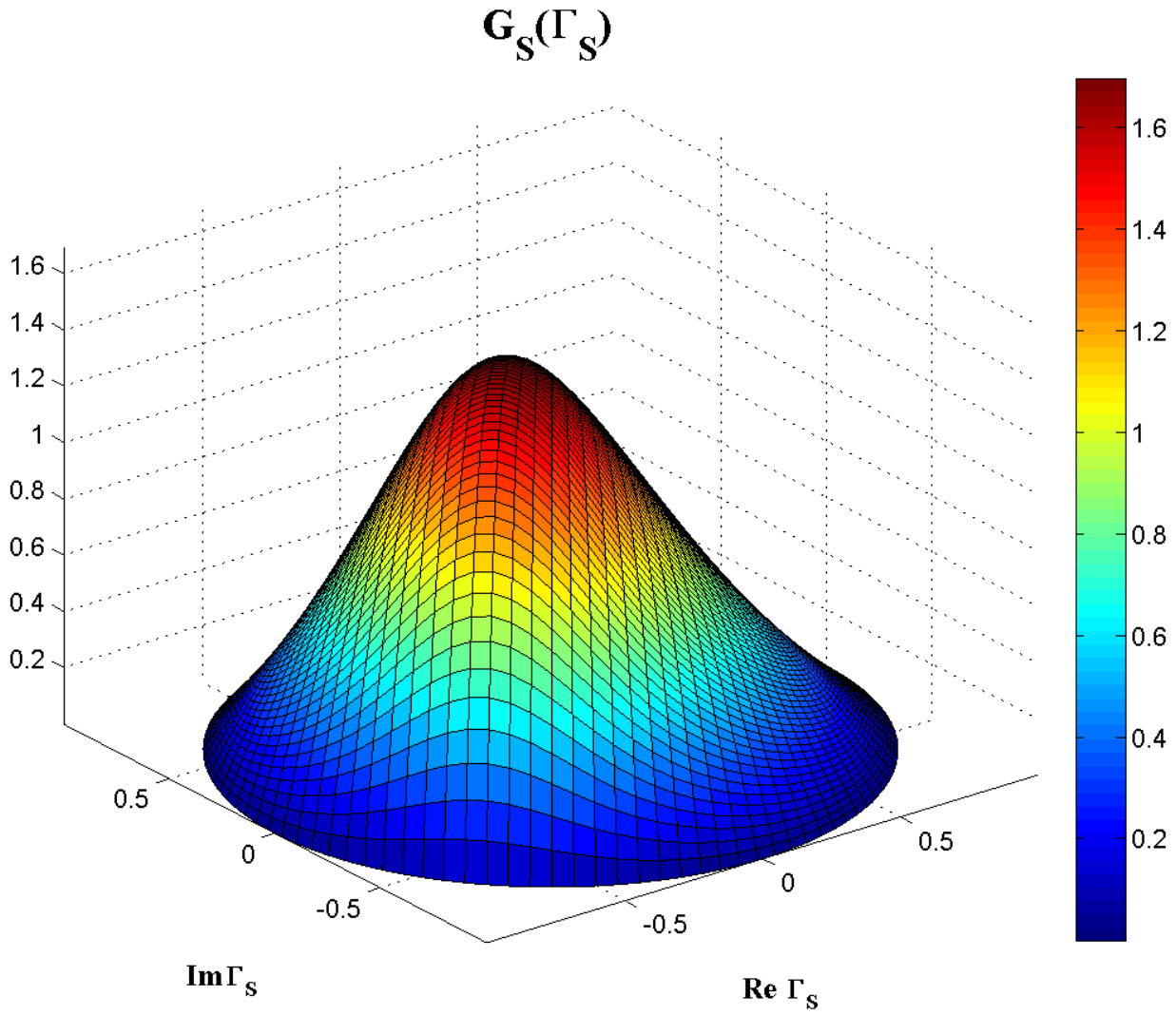
$$-0.783\ dB < G_T[dB] - G_{TU}[dB] < 0.861\ dB$$

$$G_{TU\,\max} = \frac{1}{1-|S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-|S_{22}|^2} = 17.83$$

$$G_{TU\,\max}[dB] = 12.511\ dB$$

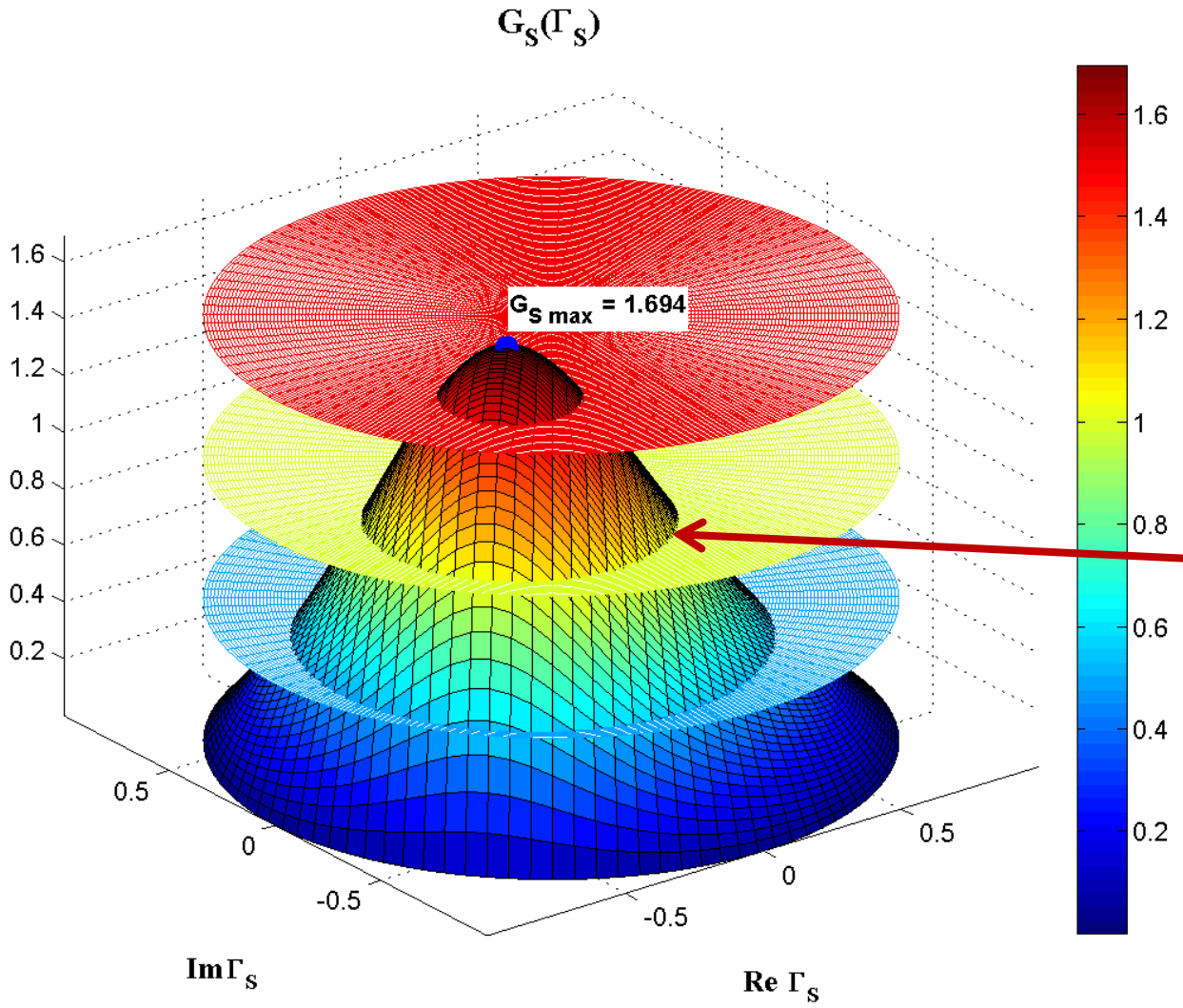
$$G_{S\,\max} = \frac{1}{1-|S_{11}|^2} = 1.694 = 2.289\ dB$$

$G_S(\Gamma_S)$



$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}$$

$G_S(\Gamma_S)$, constant value contours



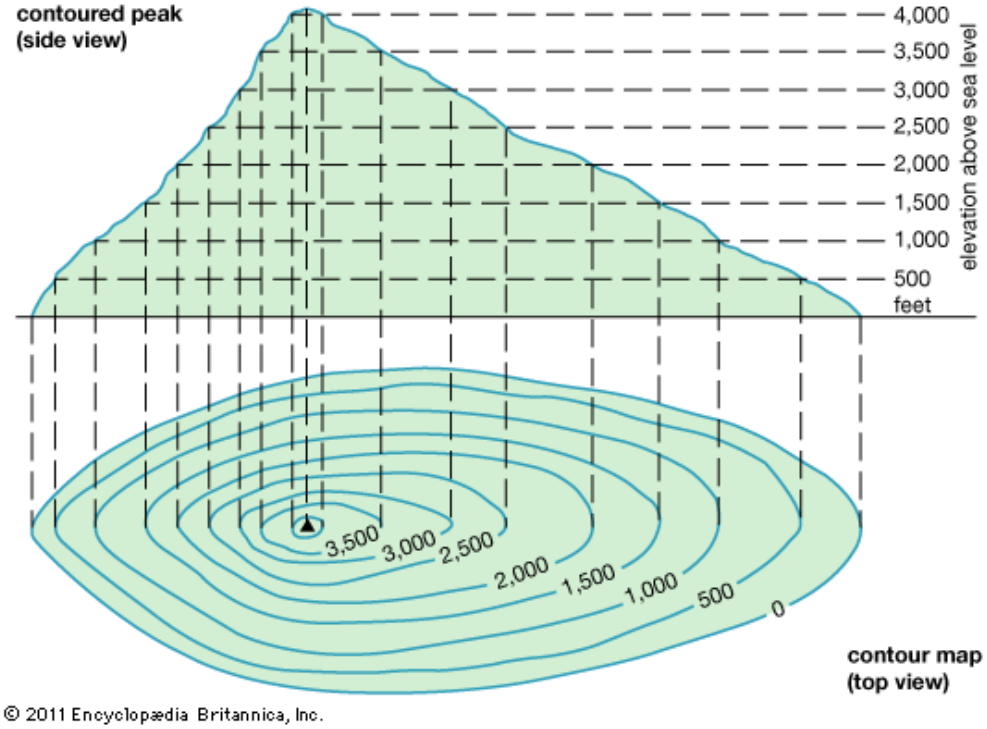
$G_S = 1.5$

$G_S = 1.0$

$G_S = 0.5$

Circles

Contour map/lines



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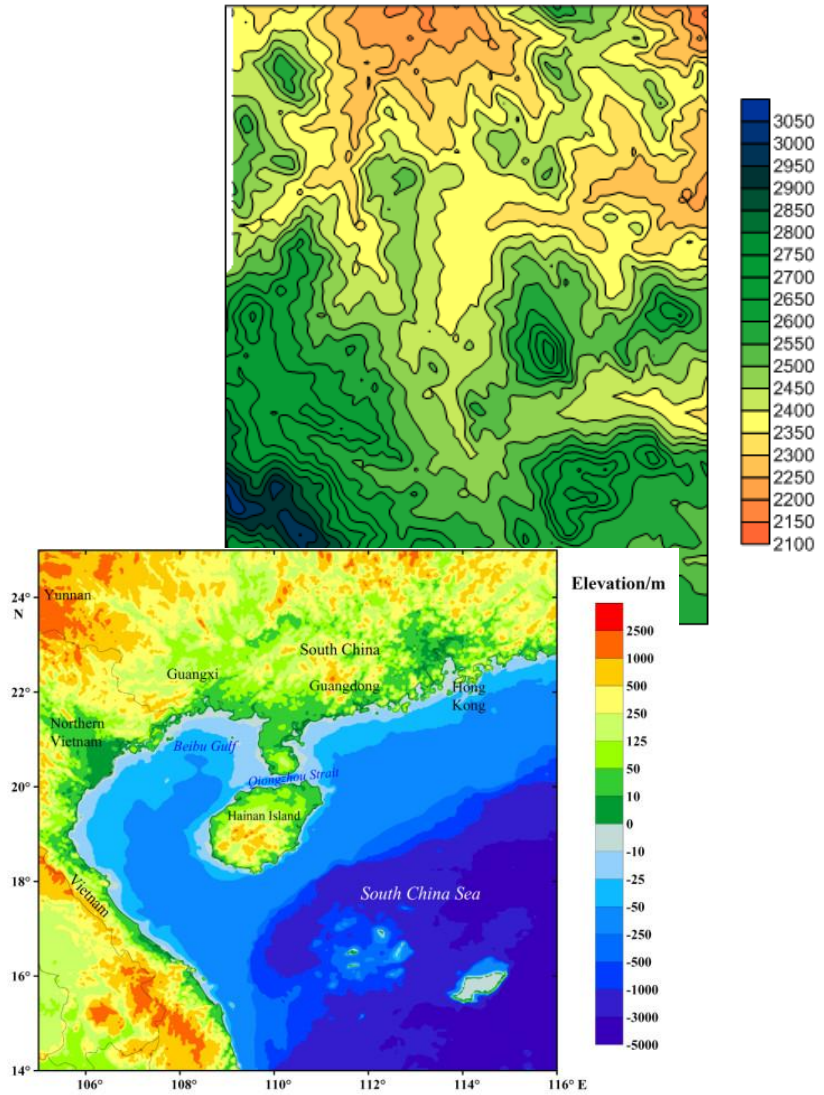
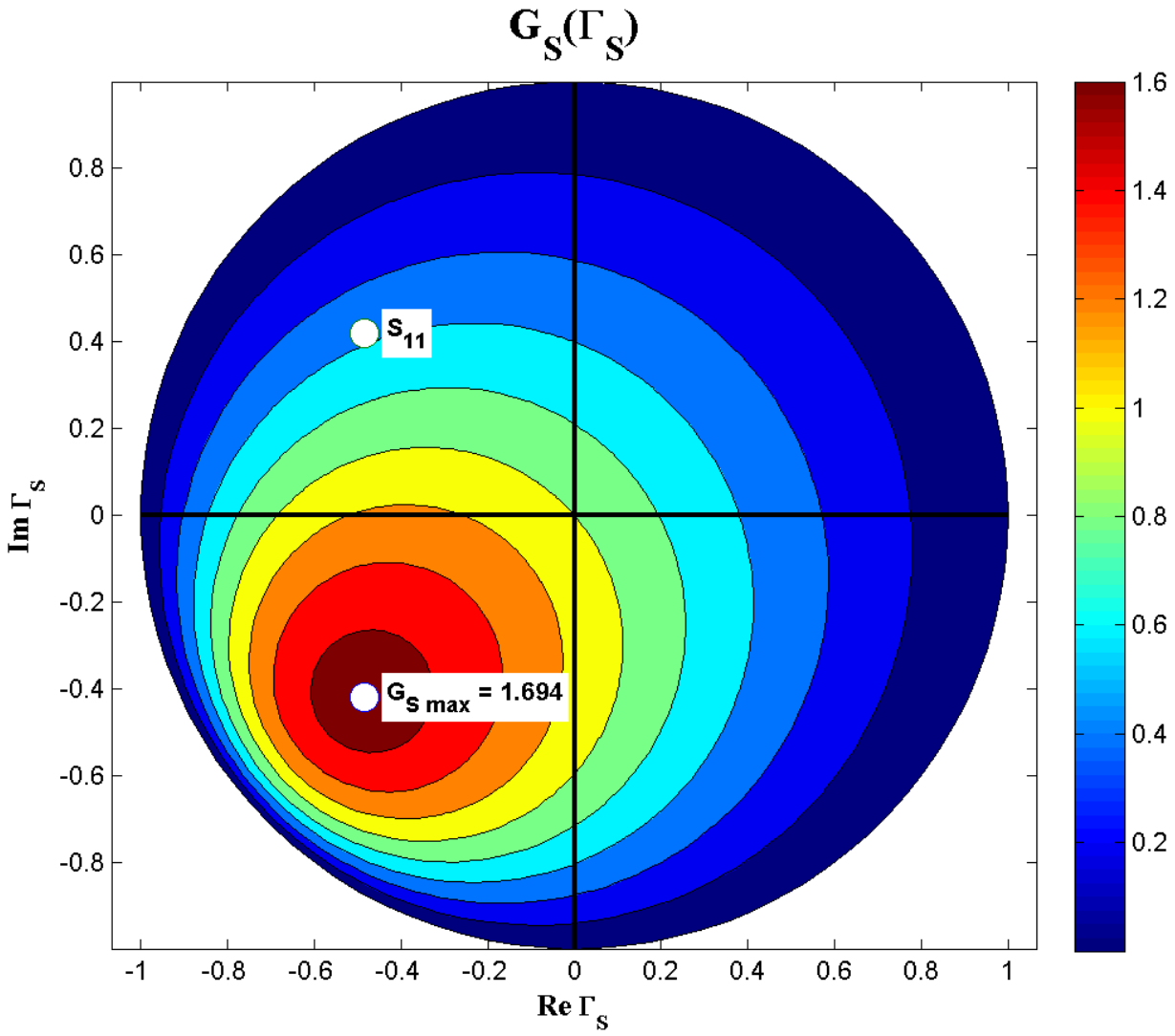


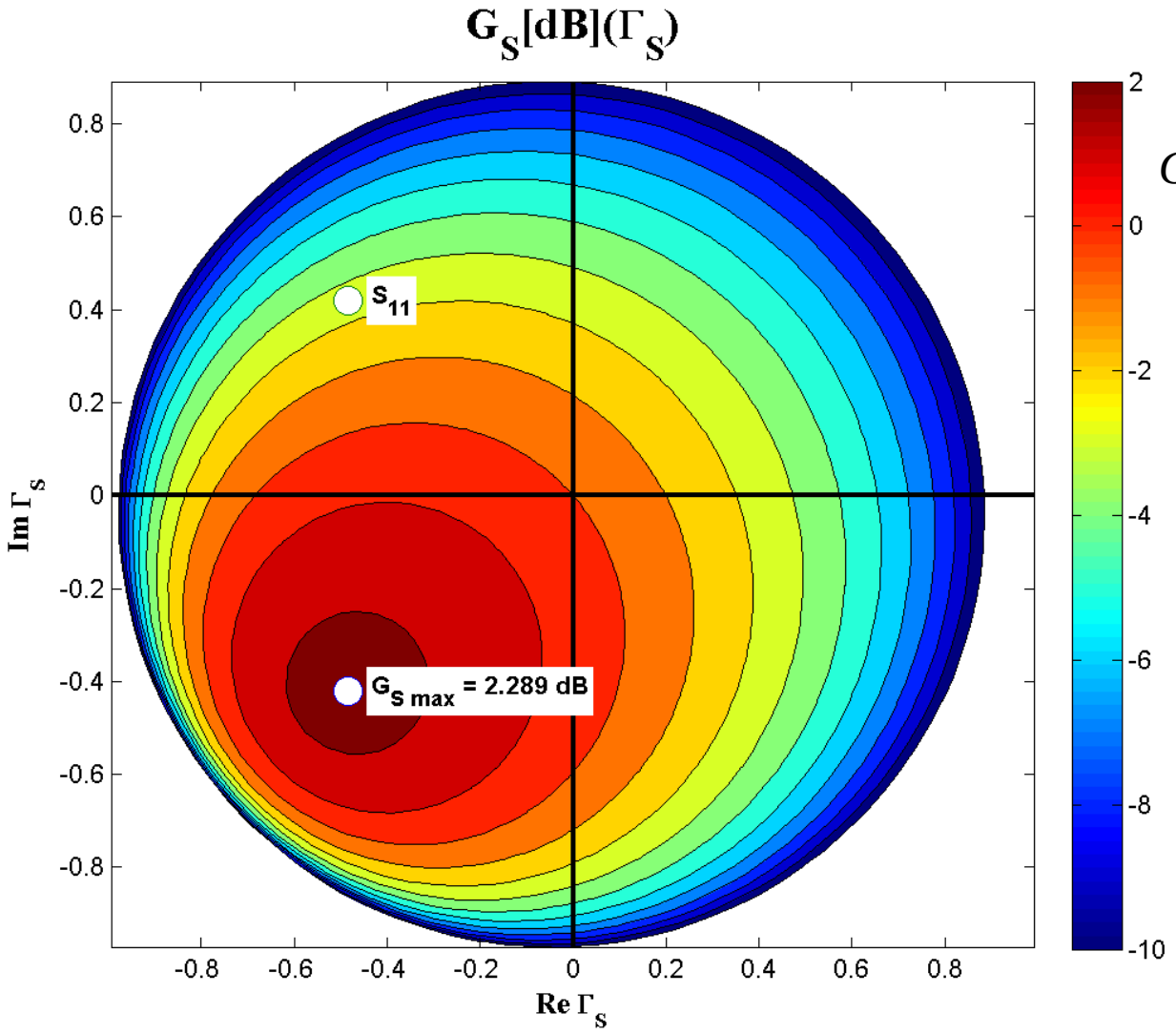
$G_S(\Gamma_S)$, constant value contours



$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}$$

$$G_{S\max} = G_S\big|_{\Gamma_S = S_{11}^*}$$

$G_S[dB](\Gamma_S)$, constant value contours



$$G_S[dB] = 10 \cdot \log\left(\frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\right)$$

$$G_{S\max} = G_S\big|_{\Gamma_S = S_{11}^*}$$

Input section constant gain circles

- The normalized gain factor (**linear scale!**)

$$g_S = \frac{G_S}{G_{S\max}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\cdot\left(1-|S_{11}|^2\right)<1$$

- Locus of the points with fixed values $g_s<1$

$$g_S\cdot|1-S_{11}\cdot\Gamma_S|^2 = \left(1-|\Gamma_S|^2\right)\cdot\left(1-|S_{11}|^2\right)$$

$$\left(g_S\cdot|S_{11}|^2+1-|S_{11}|^2\right)\cdot|\Gamma_S|^2 - g_S\cdot\left(S_{11}\cdot\Gamma_S+S_{11}^*\cdot\Gamma_S^*\right)=1-|S_{11}|^2-g_S$$

$$\Gamma_S\cdot\Gamma_S^* - \frac{g_S\cdot\left(S_{11}\cdot\Gamma_S+S_{11}^*\cdot\Gamma_S^*\right)}{1-(1-g_S)\cdot|S_{11}|^2} = \frac{1-|S_{11}|^2-g_S}{1-(1-g_S)\cdot|S_{11}|^2} \Big/ + \frac{g_S^2\cdot|S_{11}|^2}{\left[1-(1-g_S)\cdot|S_{11}|^2\right]^2}$$

Input section constant gain circles

$$\left|\Gamma_S - \frac{g_S \cdot S_{11}^*}{1-(1-g_S)\cdot|S_{11}|^2}\right| = \frac{\sqrt{1-g_S}\cdot\left(1-|S_{11}|^2\right)}{1-(1-g_S)\cdot|S_{11}|^2} \qquad |\Gamma_S - C_S| = R_S$$

$$C_S = \frac{g_S \cdot S_{11}^*}{1-(1-g_S)\cdot|S_{11}|^2} \qquad R_S = \frac{\sqrt{1-g_S}\cdot\left(1-|S_{11}|^2\right)}{1-(1-g_S)\cdot|S_{11}|^2}$$

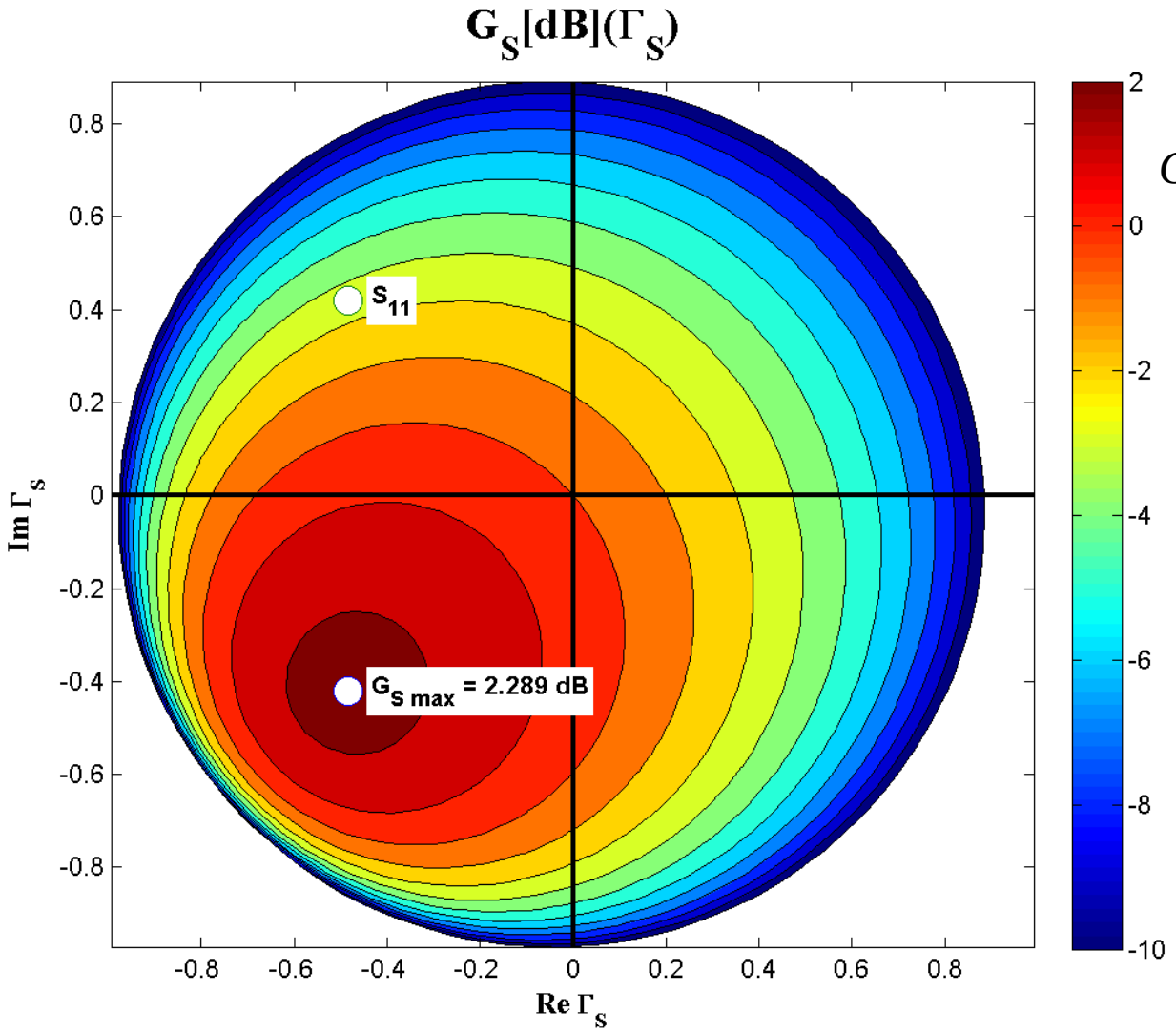
- Equation of a circle in the complex plane where Γ_S is plotted
- **Interpretation**: Any reflection coefficient Γ_S which plotted in the complex plane lies **on** the circle drawn for $g_{circle} = G_{circle}/G_{Smax}$ will lead to a gain $G_S = G_{circle}$
  - Any reflection coefficient Γ_S plotted **outside** this circle will lead to a gain $G_S < G_{circle}$
  - Any reflection coefficient Γ_S plotted **inside** this circle will lead to a gain $G_S > G_{circle}$

Input section constant gain circles

$$C_S = \frac{g_S \cdot S_{11}^*}{1-(1-g_S)\cdot|S_{11}|^2} \qquad R_S = \frac{\sqrt{1-g_S}\cdot\left(1-|S_{11}|^2\right)}{1-(1-g_S)\cdot|S_{11}|^2}$$

- The centers of each family of circles lie along straight lines given by the angle of $\Gamma_{S\max} = S_{11}^*$
- Circles are plotted (traditionally, CAD) in **logarithmic scale** ([dB])
  - formulas are in **linear scale!**
- The circle for G_S = 0 dB will always pass through the origin of the complex plane (center of the Smith chart)

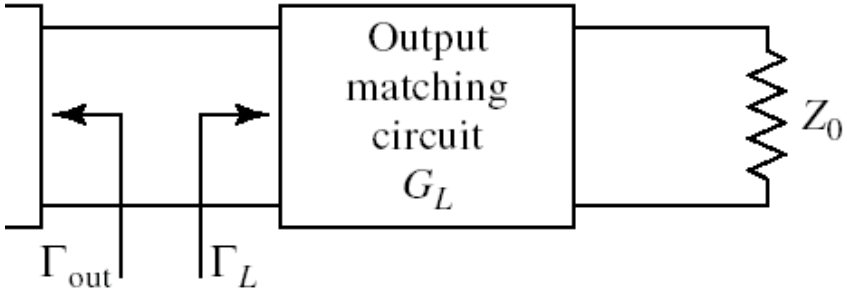
$G_S[dB](\Gamma_S)$, constant value contours



$$G_S[dB] = 10 \cdot \log\left(\frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2}\right)$$

$$G_{S\max} = G_S\big|_{\Gamma_S = S_{11}^*}$$

Output section constant gain circles



$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

- Maximum gain for $\Gamma_L = S_{22}^* \Rightarrow$ $G_{L\max} = \frac{1}{1-|S_{22}|^2}$

$$g_L = \frac{G_L}{G_{L\max}} = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}\cdot\left(1-|S_{22}|^2\right)<1$$

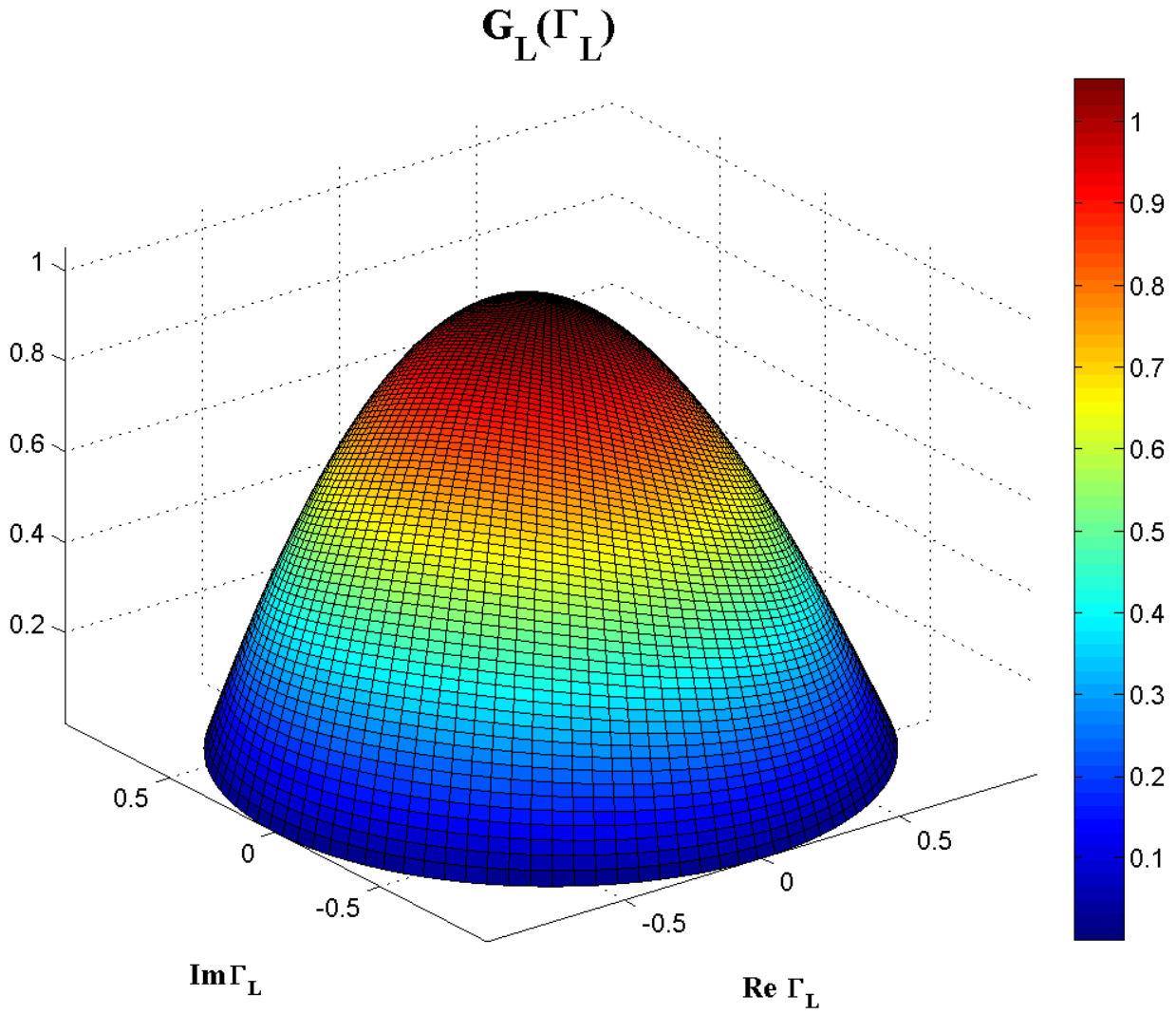
- Similar computations

$$C_L = \frac{g_L\cdot S_{22}^*}{1-(1-g_L)\cdot|S_{22}|^2} \qquad R_L = \frac{\sqrt{1-g_L}\cdot\left(1-|S_{22}|^2\right)}{1-(1-g_L)\cdot|S_{22}|^2}$$

- Example

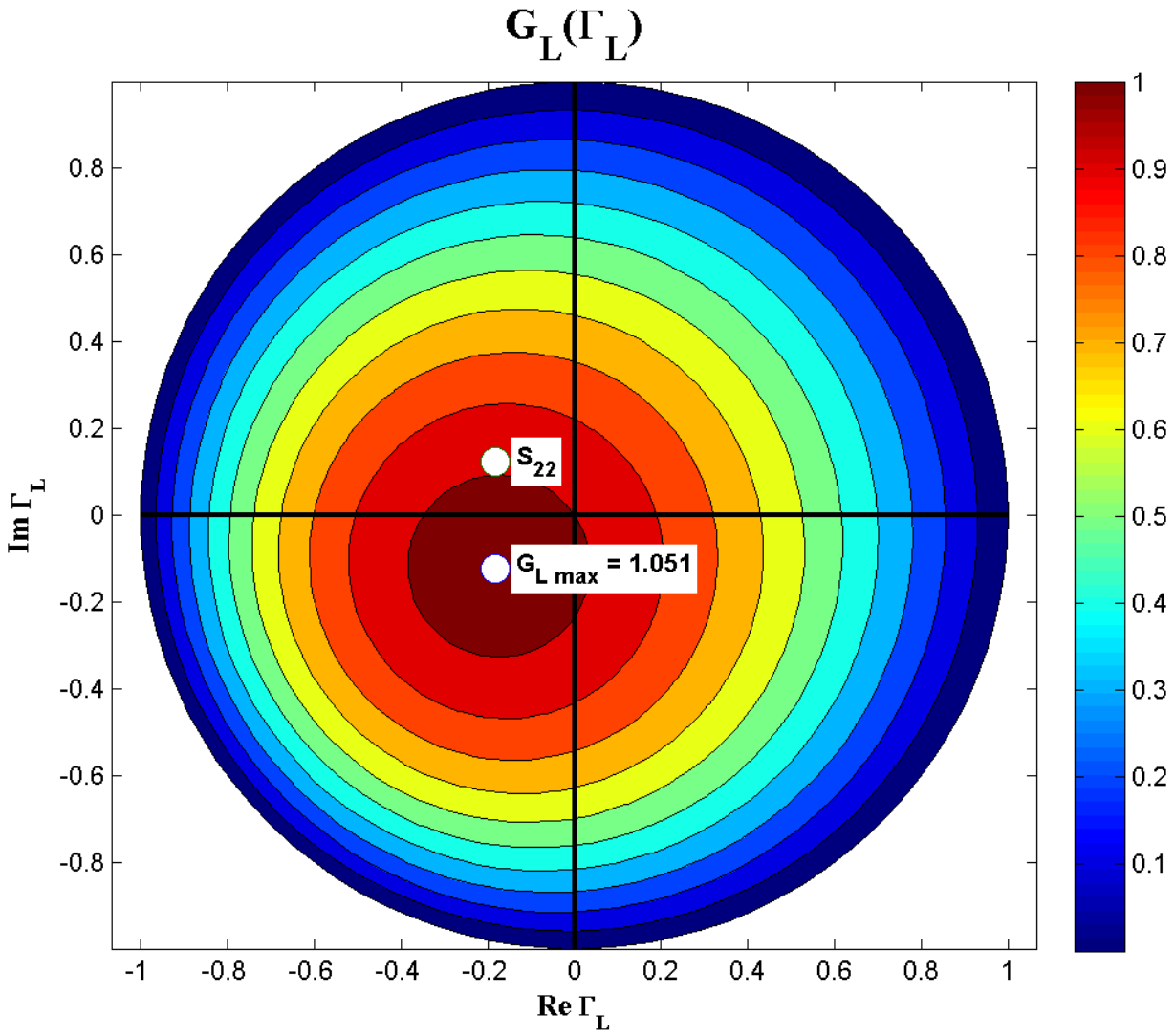
$$G_{L\max} = \frac{1}{1-|S_{22}|^2} = 1.051 = 0.215\,dB$$

$G_L(\Gamma_L)$



$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

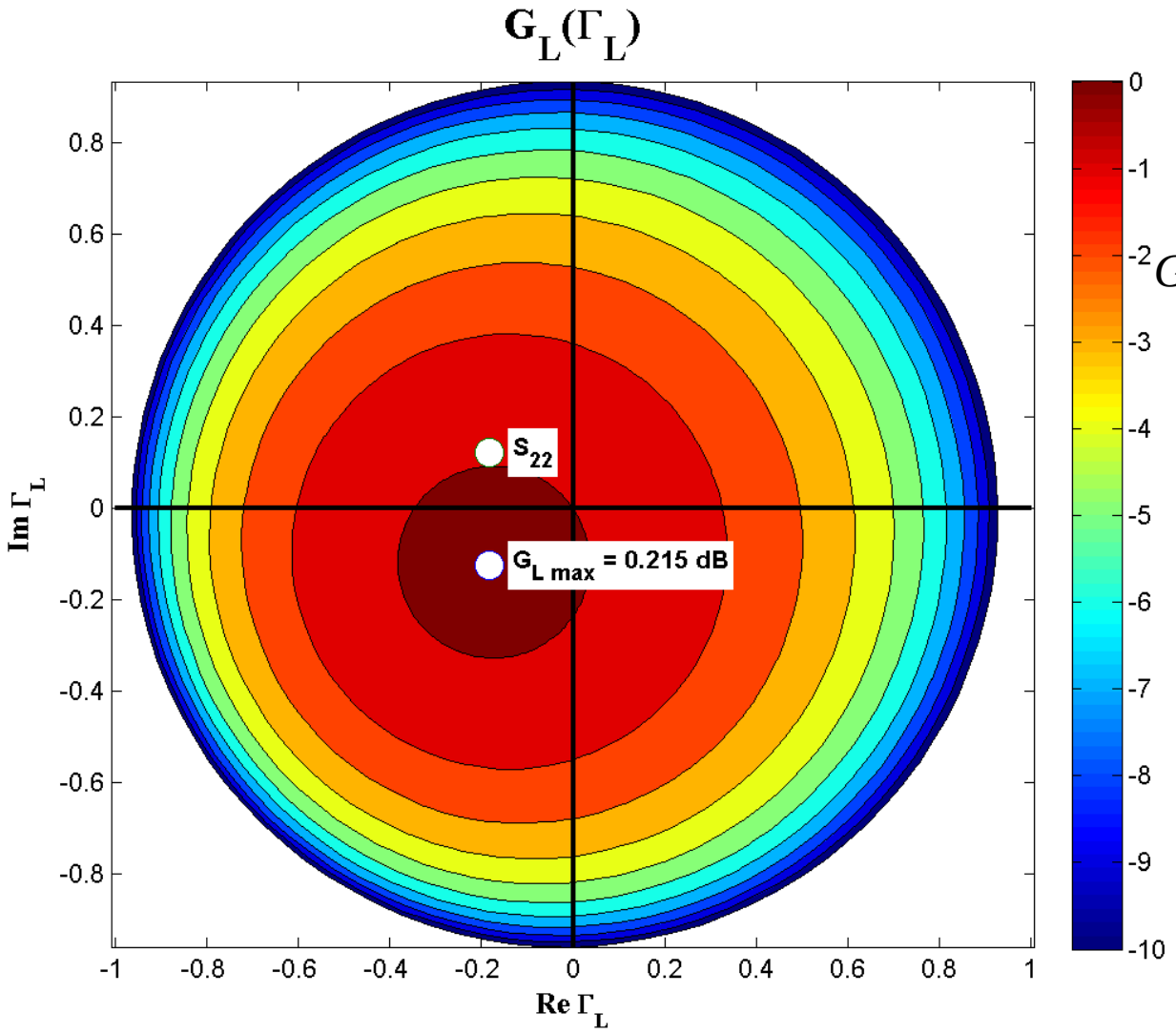
$G_L(\Gamma_L)$, constant value contours



$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$G_{L\max} = G_L\big|_{\Gamma_L = S_{22}^*}$$

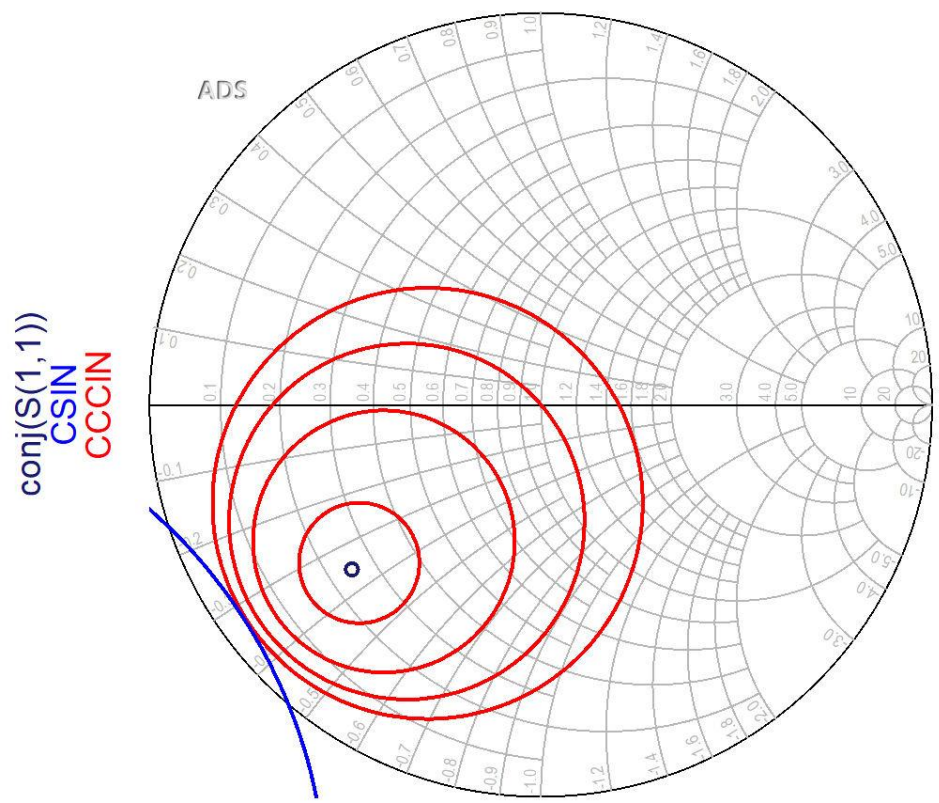
$G_L[dB](\Gamma_L)$, constant value contours

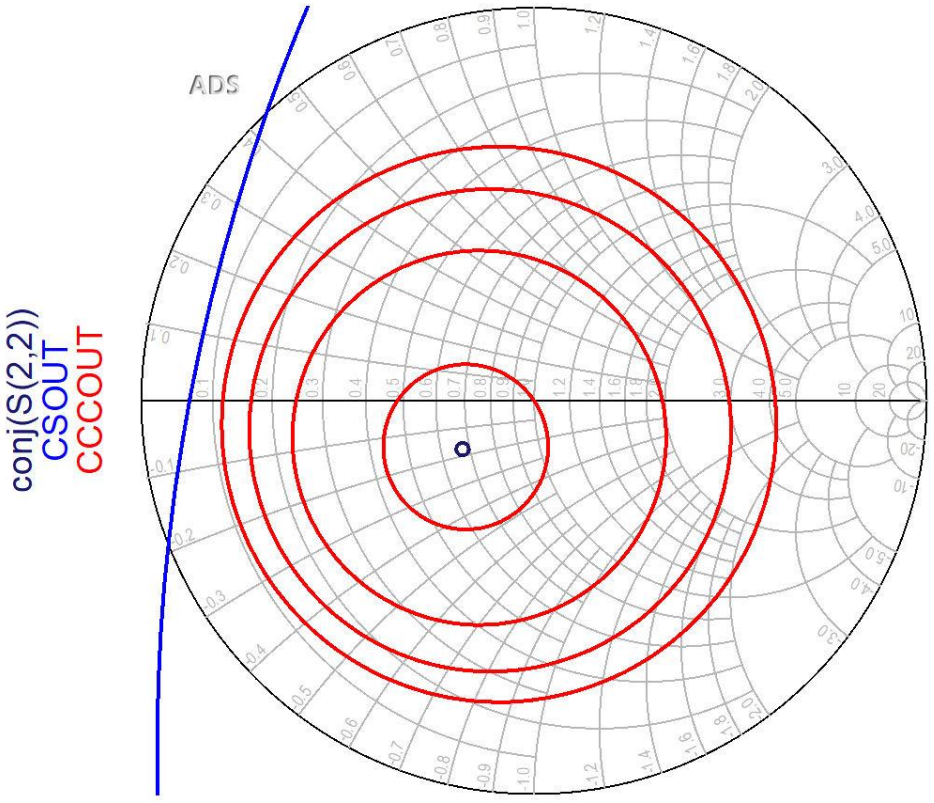


$$G_L[dB] = 10 \cdot \log\left(\frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}\right)$$

$$G_{L\max} = G_L\big|_{\Gamma_L = S_{22}^*}$$

ADS

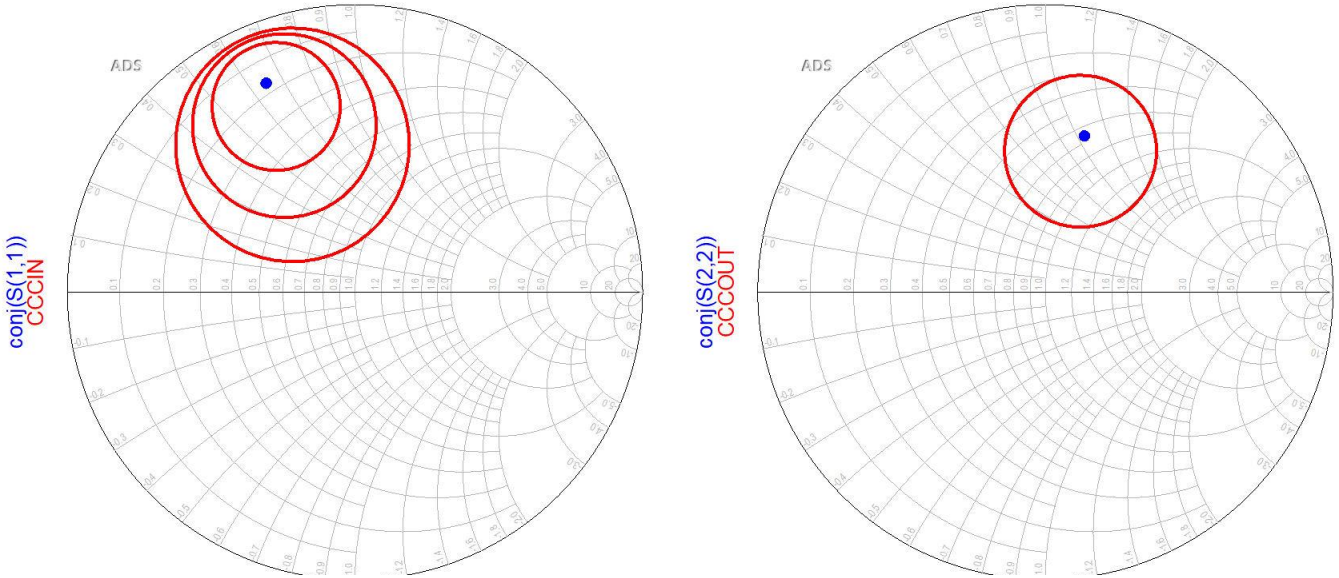




- Circles are plotted for requested values (**in dB!**)
- It is usefull to compute G_{Smax} and G_{Lmax} before
  - in order to request relevant circles

Schematic 1 – Lab 3

- **multiple circles (families)** are plotted and some required values are computed



freq	K	MAG	NFmin	Sopt	Rn	G0	GLmax	GSmax
5.000 GHz	0.533	15.296	0.700	0.660 / 106....	19.500	8.974	1.634	4.249

Design for Specified Gain

- We compute G_o, G_{Smax} , G_{Lmax}
- To obtain the design gain we **choose** supplemental gain needed (supplemental to constant G_o)
  - we account for the deviation that might arise from the unilateral assumption (using unilateral figure of merit U)

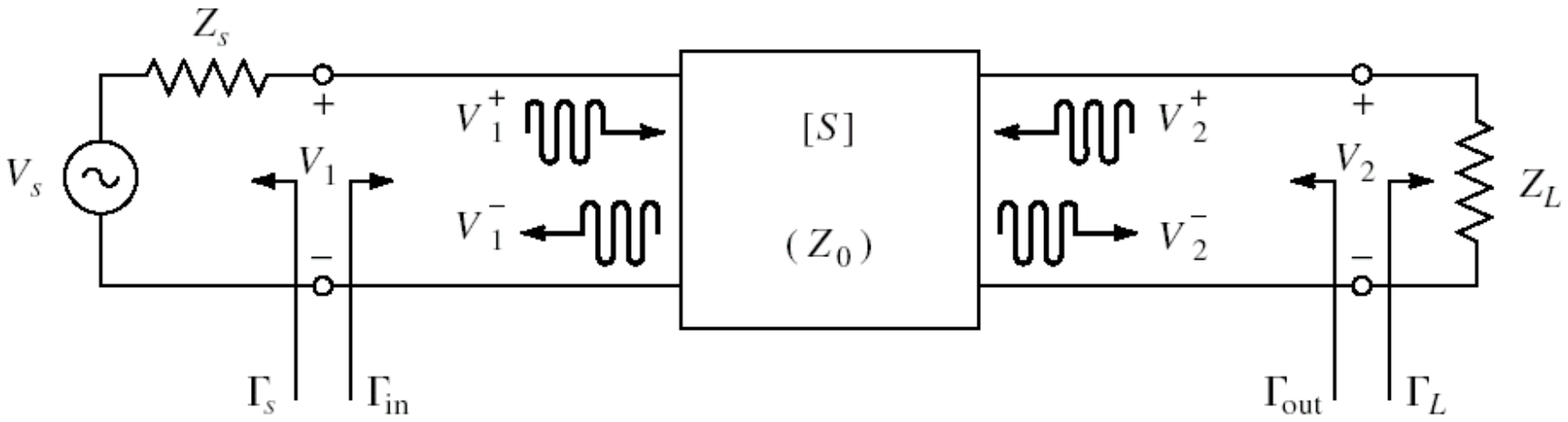
$$G_{design}[dB] = G_{S_design}[dB] + G_0[dB] + G_{L_design}[dB]$$

- We plot the circles for design (chosen) values G_{S_design} , G_{L_design}
- We design input and output matching circuits which move the reflection coefficient **on** or **inside** the design circles (depending on specific application requirements)

Microwave Amplifiers

Low-Noise Amplifier Design

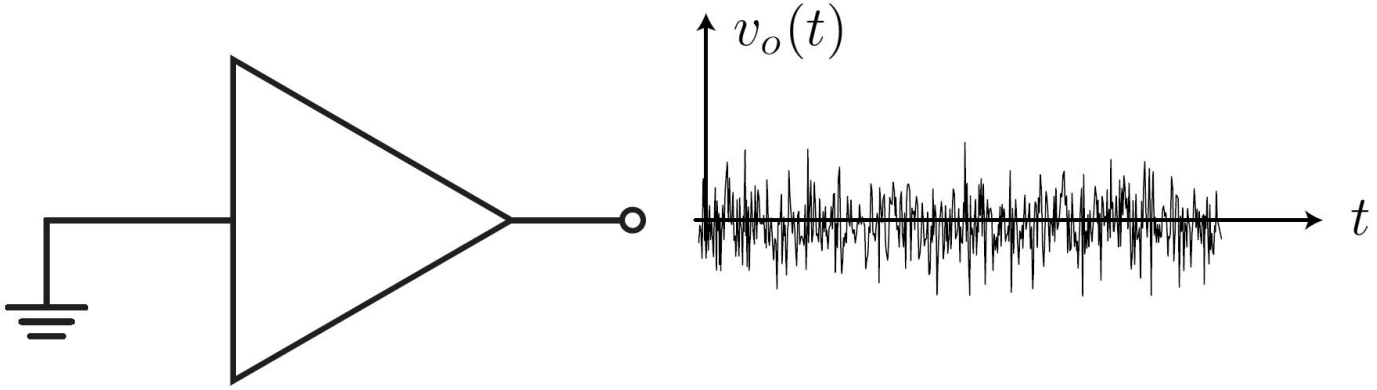
Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - power gain
  - **noise** (sometimes – **small signals**)
  - linearity (sometimes – large signals)

Noise

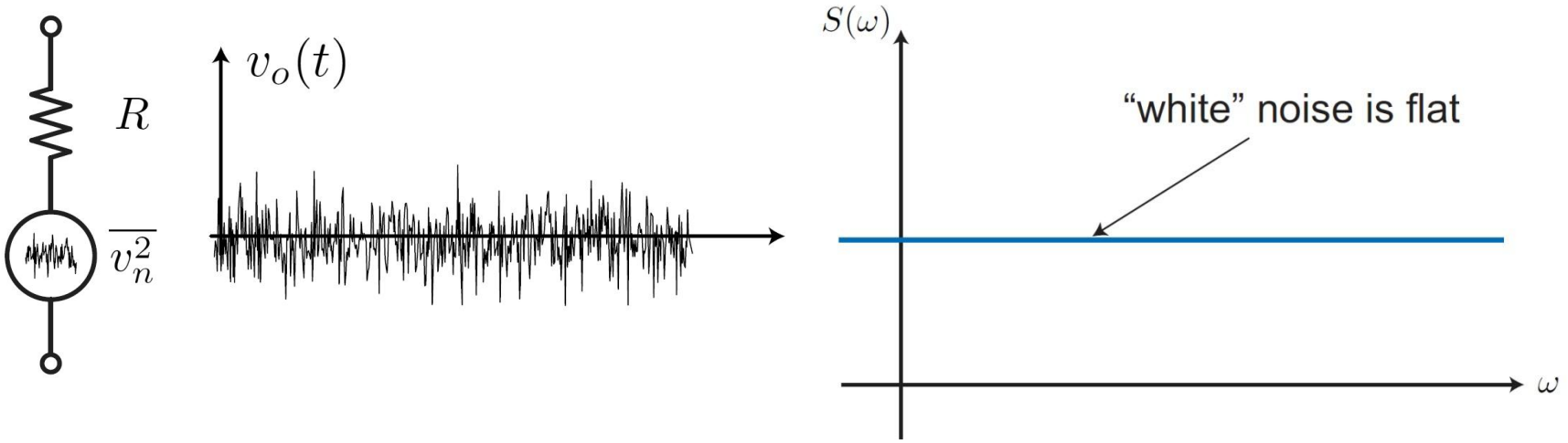
- Noise: random fluctuations of the signal



$$\overline{v_n(t)} = \langle v_n(t) \rangle = \frac{1}{T}\int_0^T v_n(t)\,dt = 0$$

$$\overline{v_n^2(t)} = \langle v_n^2(t) \rangle = \frac{1}{T}\int_0^T v_n^2(t)\,dt \neq 0 \qquad V_{n(ef)} = \sqrt{\overline{v_n^2(t)}}$$

Noise



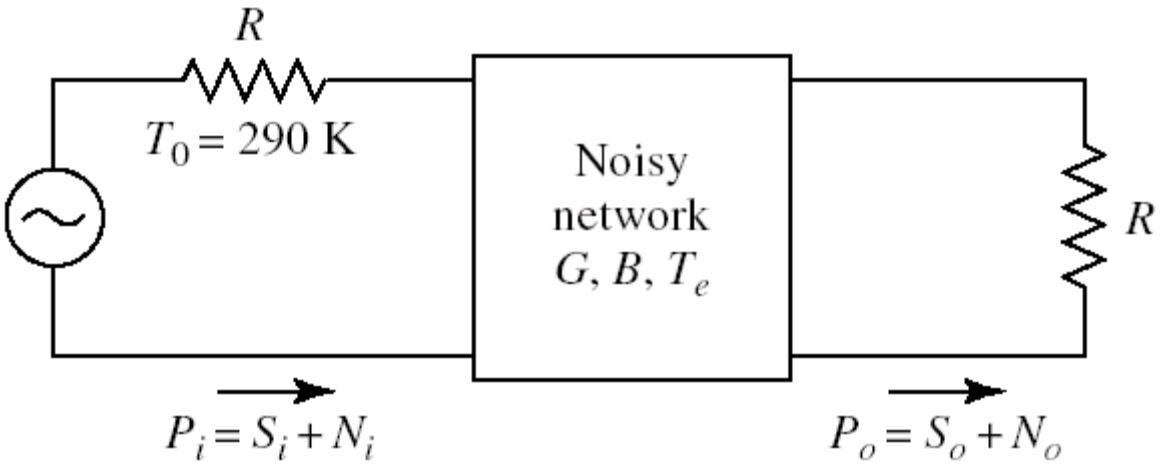
- effective noise voltage

$$V_{n(ef)} = \sqrt{4kTBR}$$

- noise power available (for maximum power transfer with impedance/resistance matching)

$$P_n = kTB$$

Noise Figure F



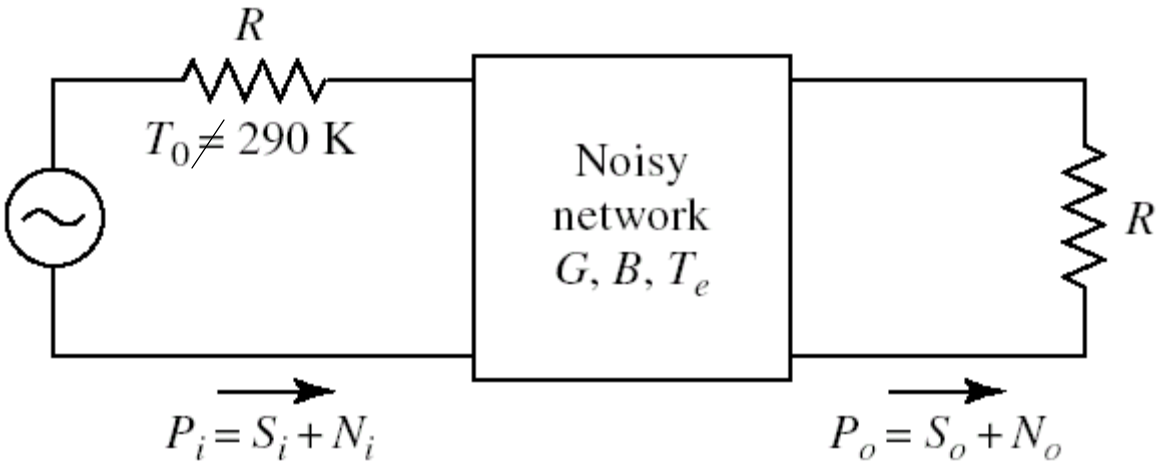
- The noise figure F, is a measure of the reduction in signal-to-noise ratio between the input and output of a device, when (by definition) the input noise power is assumed to be the noise power resulting from a matched resistor at To = 290 K (reference noise conditions)

$$F = \left. \frac{S_i / N_i}{S_o / N_o} \right|_{T_0 = 290K}$$

$$V_{n(ef)} = \sqrt{4kTBR}$$

$$P_n = kTB$$

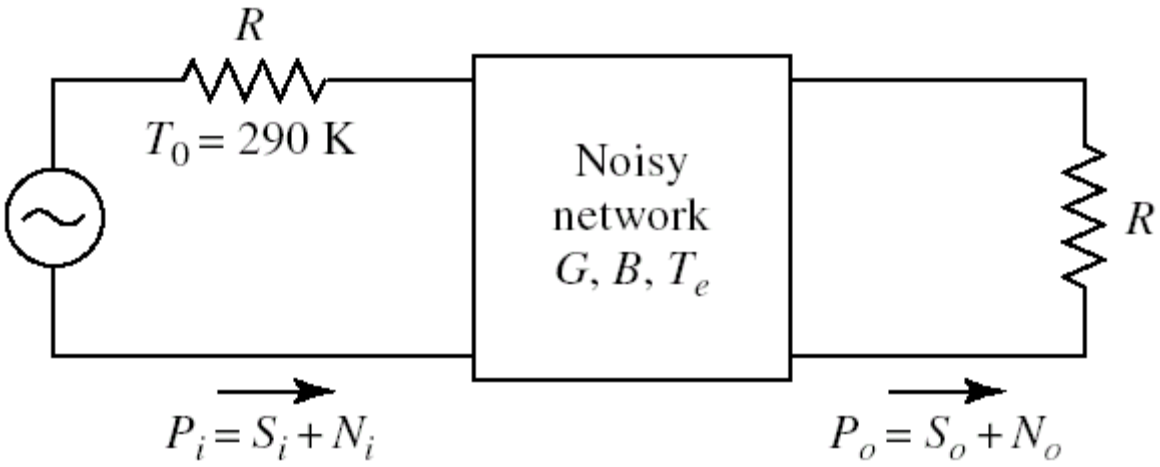
Noise Figure F



- The noise figure F, **is not** directly a measure of the reduction in signal-to-noise ratio between the input and output of a device, when the input noise power is different from that of the reference noise conditions

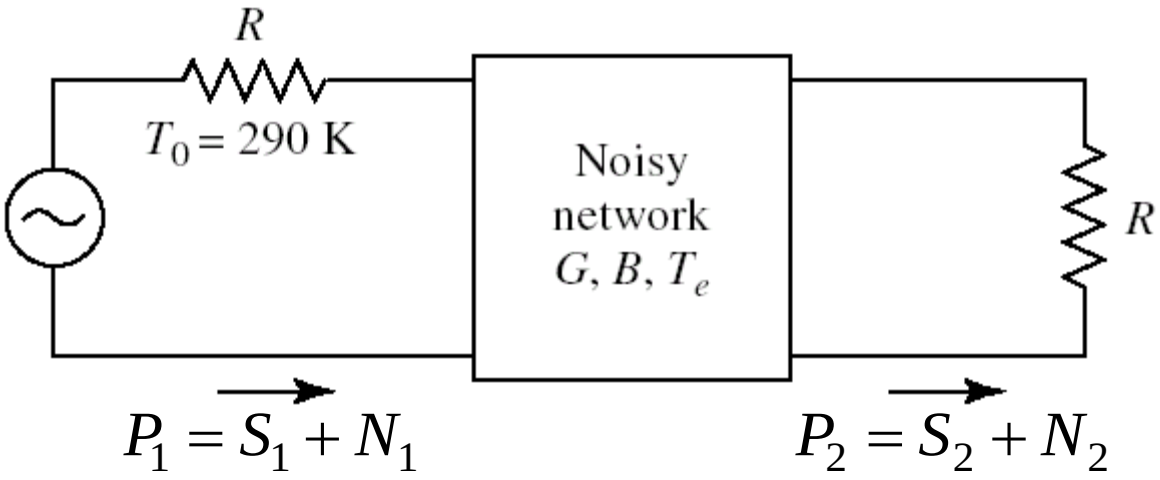
$$F \neq \left. \frac{S_i / N_i}{S_o / N_o} \right|_{T_0 \neq 290K}$$

Noise Figure F



- In general, the output noise power consists of two elements:
  - the input noise power amplified or attenuated by the device (for example amplified with the power gain G applied also to the desired signal)
  - a noise power generated internally by the network if the network is noisy (this power **does not** depend on the input noise power)

Noise Figure F



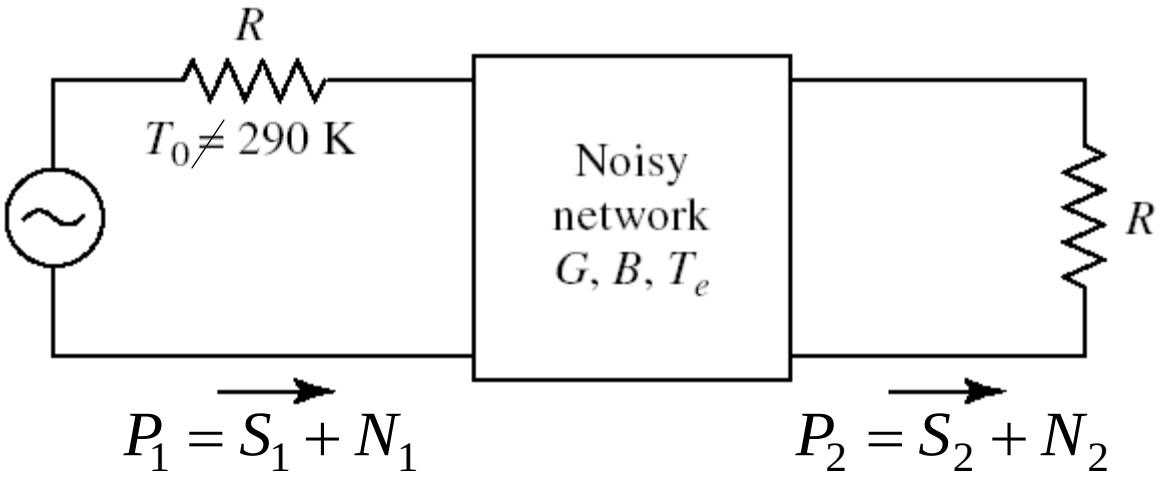
- Estimation of the internally generated noise power can be done using the Noise Figure F definition:

$$F = \left.\frac{S_1/N_1}{S_2/N_2}\right|_{T_0=290K, N_1=N_0}$$

$$N_2 = F \cdot N_0 \cdot \frac{S_2}{S_1} = F \cdot N_0 \cdot G$$

$$N_2 = N_0 \cdot G + (F-1) \cdot N_0 \cdot G$$

Noise Figure F

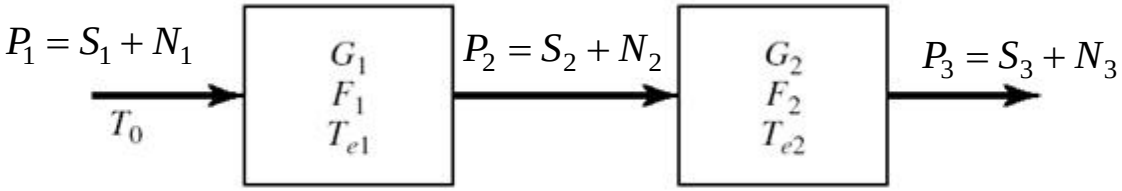


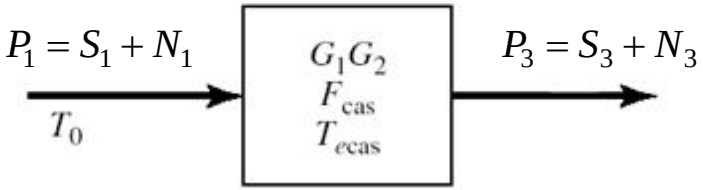
- We identify the two terms:
  - amplified input noise
  - internally generated noise
- When the input noise does not correspond to reference noise conditions (N1 ≠ No)
  - the internally generated noise does not change

$$N_2 = N_0 \cdot G + (F-1) \cdot N_0 \cdot G$$

$$N_2 = N_1 \cdot G + (F-1) \cdot N_0 \cdot G$$

Noise figure of a cascaded system





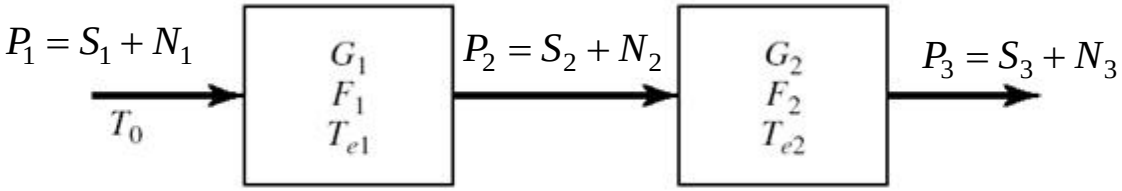
$$N_2 = N_1 \cdot G_1 + (F_1 - 1) \cdot N_0 \cdot G_1 \qquad G_{cas} = G_1 \cdot G_2$$

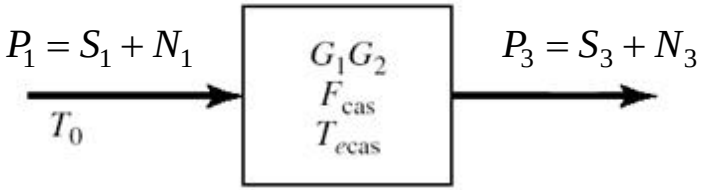
$$N_3 = N_2 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2 \qquad N_3 = N_1 \cdot G_{cas} + (F_{cas} - 1) \cdot N_0 \cdot G_{cas}$$

$$N_3 = [N_1 \cdot G_1 + (F_1 - 1) \cdot N_0 \cdot G_1] \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

$$N_3 = N_1 \cdot G_1 \cdot G_2 + (F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

Noise figure of a cascaded system





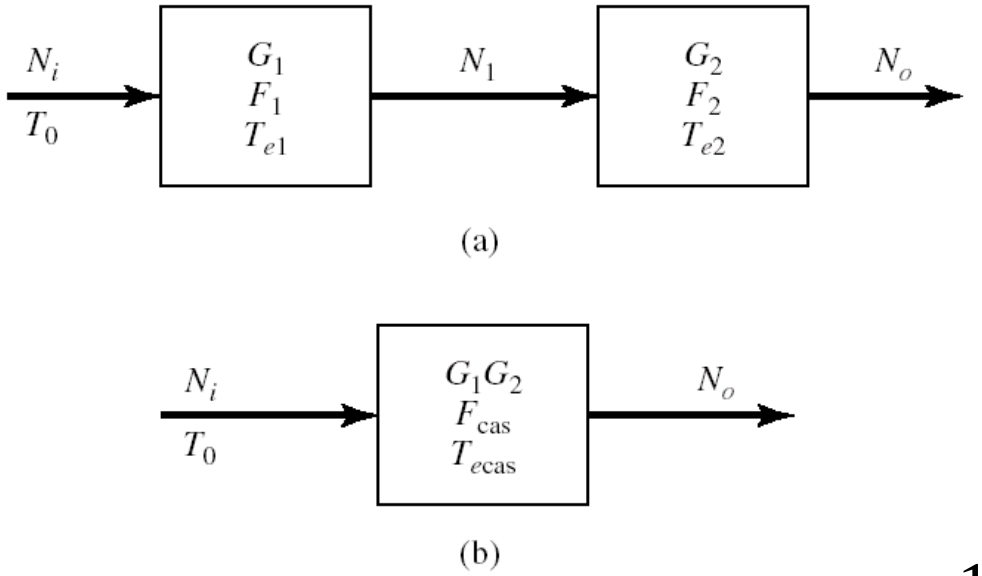
$$N_3 = N_1 \cdot G_1 \cdot G_2 + (F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2$$

$$G_{cas} = G_1 \cdot G_2 \qquad N_3 = N_1 \cdot G_{cas} + (F_{cas} - 1) \cdot N_0 \cdot G_{cas}$$

$$(F_1 - 1) \cdot N_0 \cdot G_1 \cdot G_2 + (F_2 - 1) \cdot N_0 \cdot G_2 = (F_{cas} - 1) \cdot N_0 \cdot G_1 \cdot G_2$$

$$F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1)$$

Noise figure of a cascaded system



$$G_{cas} = G_1 \cdot G_2 \qquad F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1)$$

- Friis Formula (**!linear scale**)

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \cdots$$

Friis Formula (noise)

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \cdots$$

- Friis Formula shows that:
  - the overall noise figure of a cascaded system is largely determined by the noise characteristics of the first stage
  - the noise introduced by the following stages is reduced:
    - -1
    - division by G (usually G > 1)

Friis Formula (noise)

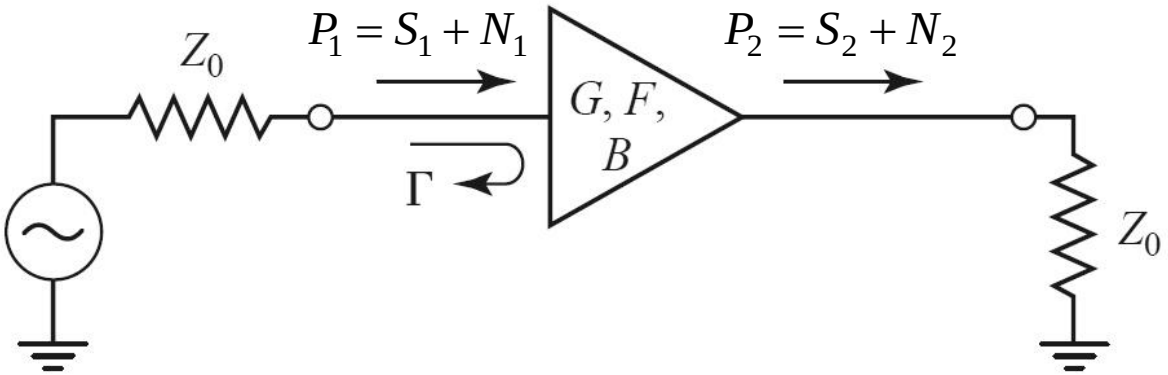
$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} + \frac{F_4 - 1}{G_1 \cdot G_2 \cdot G_3} + \cdots$$

- Effects of Friis Formula:
- in multi stage amplifiers:
  - it's essential that the first stage is as noiseless as possible even if that means sacrificing power gain
  - the following stages can be optimized for power gain
- in single stage amplifiers:
  - in the input matching circuit it's important to have noiseless elements (pure reactance, lossless lines)
  - output matching circuit has less influence on the noise (noise generated at this level appears when the desired signal has already been amplified by the transistor)

$$V_{n(ef)} = \sqrt{4kTBR} \qquad P_n = kTB$$

Noise Figure of a Mismatched Amplifier

- An input mismatched amplifier($\Gamma \neq 0$)



$$G_{ech} = G \cdot \left(1 - |\Gamma|^2\right)$$

$$N_2 = N_1 \cdot G \cdot \left(1 - |\Gamma|^2\right) + (F-1) \cdot N_0 \cdot G = N_1 \cdot G \cdot \left(1 - |\Gamma|^2\right) + \frac{F-1}{1-|\Gamma|^2} \cdot N_0 \cdot G \cdot \left(1 - |\Gamma|^2\right)$$

$$N_2 = N_1 \cdot G_{ech} + (F_{ech} - 1) \cdot N_0 \cdot G_{ech}$$

$$F_{ech} = 1 + \frac{F-1}{1-|\Gamma|^2} \geq F$$

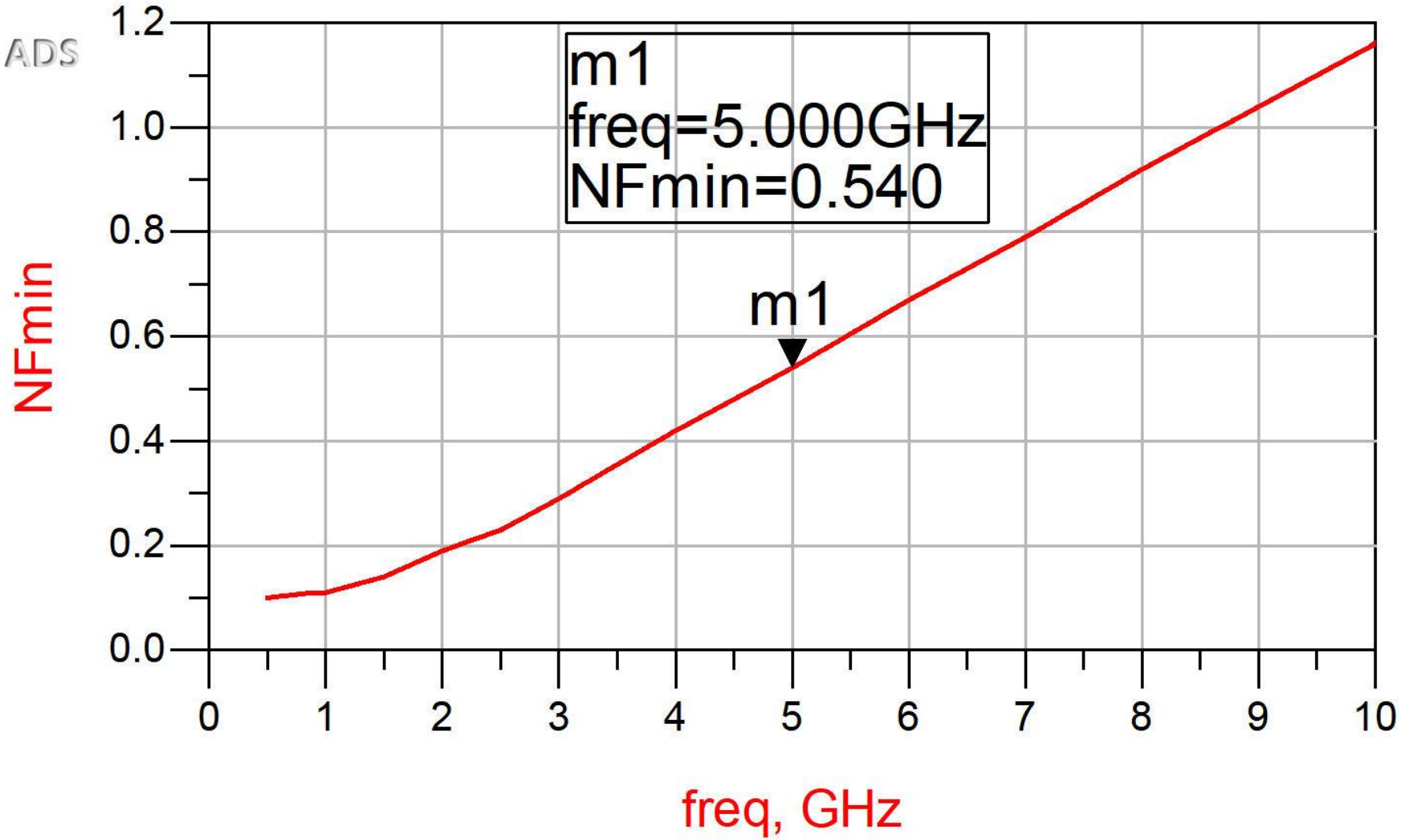
- Good noise figure **requires** good impedance matching

Example

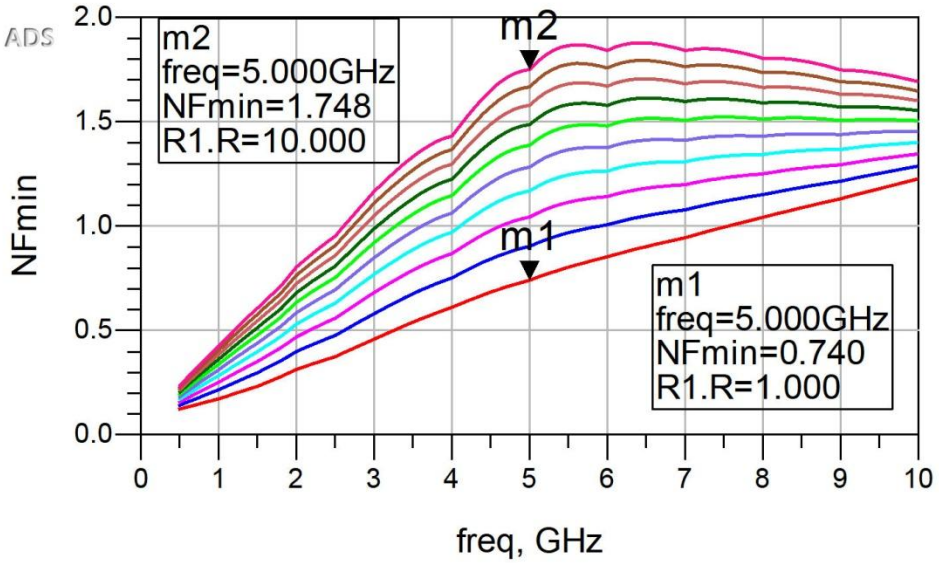
- ATF-34143 **at Vds=3V Id=20mA.**
- @5GHz
  - S11 = 0.64∠139°
  - S12 = 0.119∠-21°
  - S21 = 3.165 ∠16°
  - S22 = 0.22 ∠146°
  - Fmin = 0.54 (**tipic [dB]**
  - Γ_{opt} = 0.45 ∠174°
  - r_n = 0.03

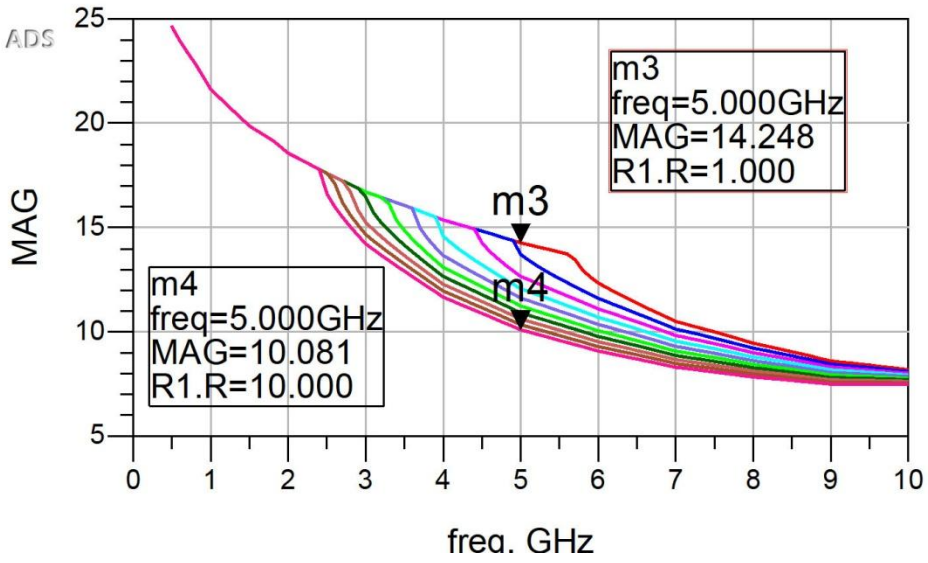
```
!ATF-34143
!S-PARAMETERS at Vds=3V  Id=20mA.   LAST UPDATED 01-29-99

# ghz s ma r 50

2.0  0.75  -126  6.306  90  0.088  23  0.26  -120
2.5  0.72  -145  5.438  75  0.095  15  0.25  -140
3.0  0.69  -162  4.762  62  0.102  7  0.23  -156
4.0  0.65  166  3.806  38  0.111  -8  0.22  174
5.0  0.64  139  3.165  16  0.119  -21  0.22  146
6.0  0.65  114  2.706  -5  0.125  -35  0.23  118
7.0  0.66  89  2.326  -27  0.129  -49  0.25  91
8.0  0.69  67  2.017  -47  0.133  -62  0.29  67
9.0  0.72  48  1.758  -66  0.135  -75  0.34  46

!FREQ   Fopt   GAMMA OPT      RN/Zo
!GHZ    dB     MAG    ANG     -

2.0  0.19  0.71  66  0.09
2.5  0.23  0.65  83  0.07
3.0  0.29  0.59  102  0.06
4.0  0.42  0.51  138  0.03
5.0  0.54  0.45  174  0.03
6.0  0.67  0.42  -151  0.05
7.0  0.79  0.42  -118  0.10
8.0  0.92  0.45  -88  0.18
9.0  1.04  0.51  -63  0.30
10.0  1.16  0.61  -43  0.46
```

Example

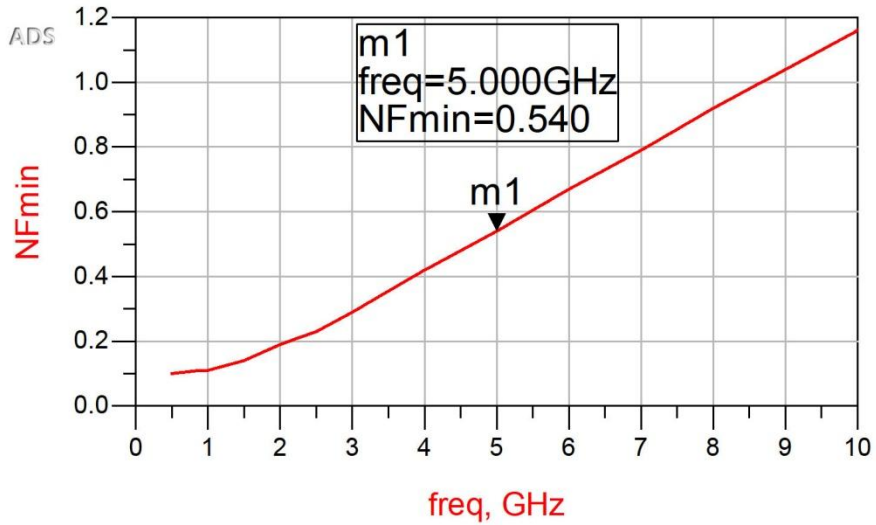

ADS
m1
freq=5.000GHz
NFmin=0.540
m1
NFmin
freq, GHz
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
7
8
9
10

Stabilization, input series resistor

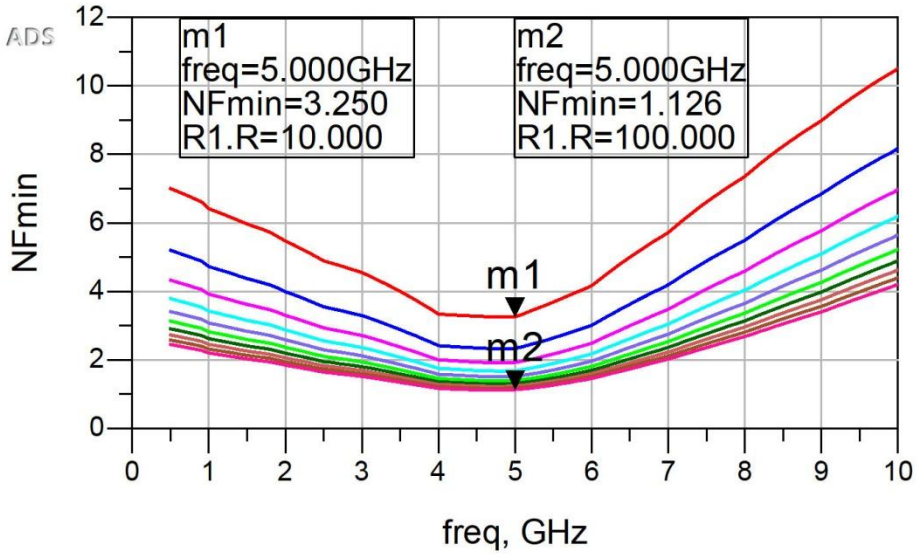


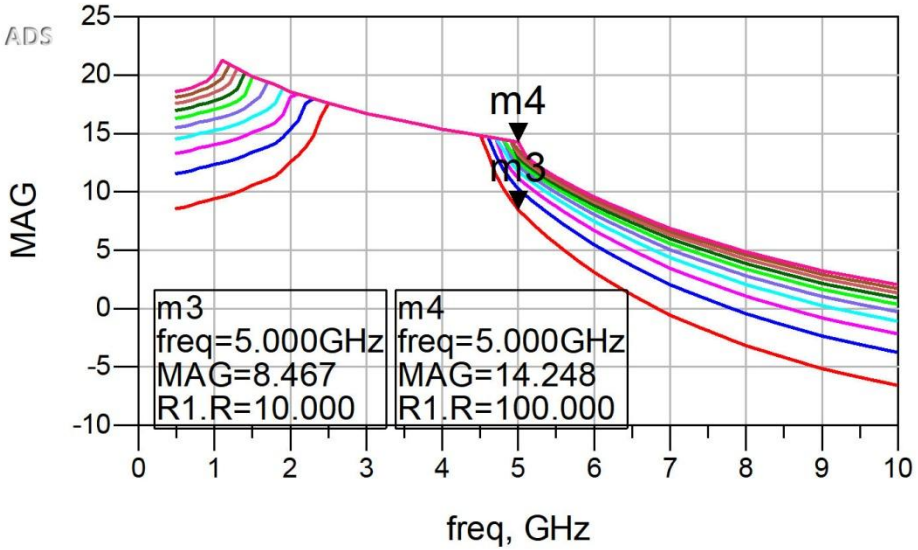


$$R_{SS} = 1 \div 10\ \Omega$$

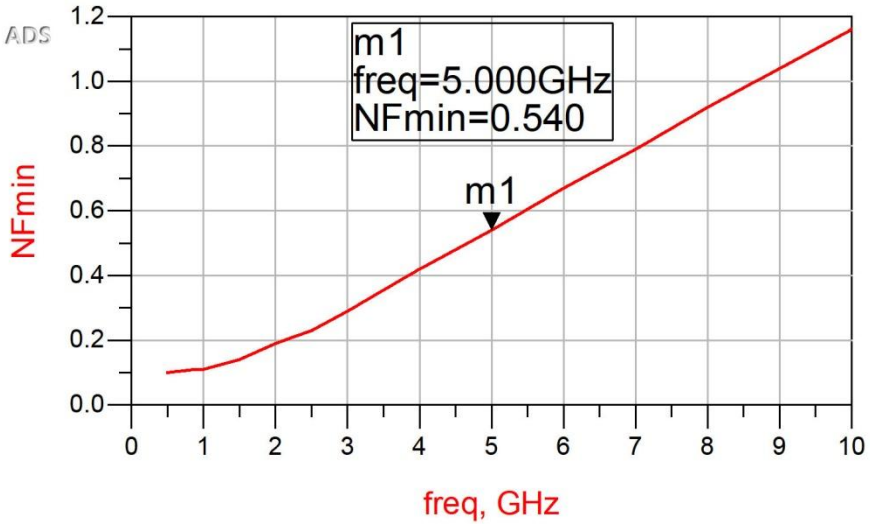


Stabilization, input shunt resistor

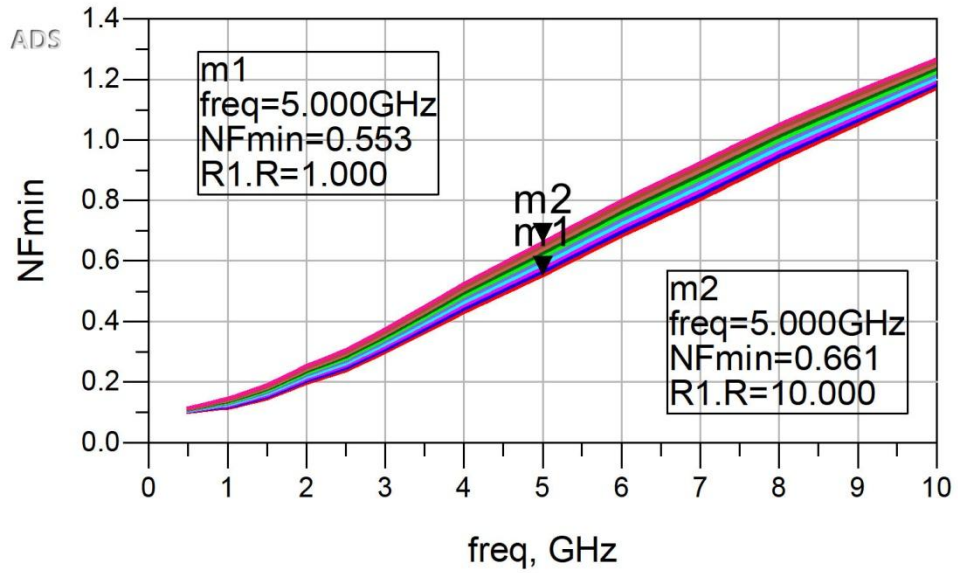


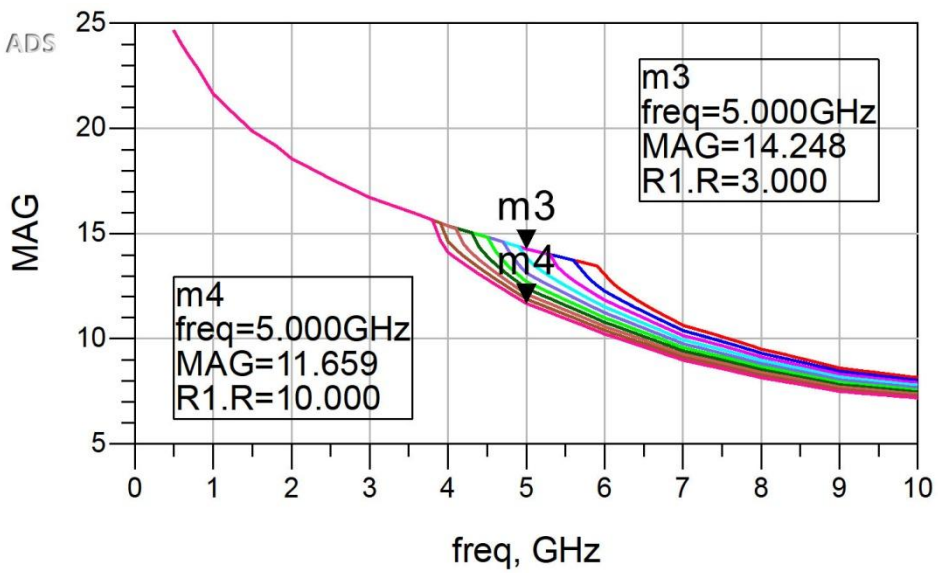


$$R_{PS} = 10 \div 100\ \Omega$$

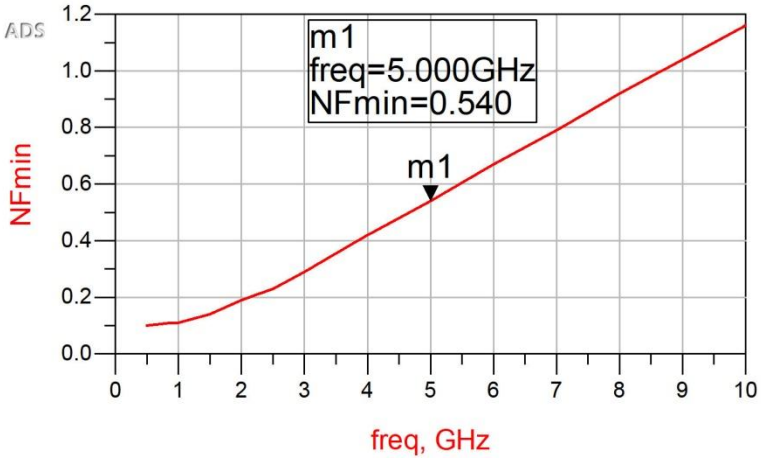


Stabilization, output series resistor

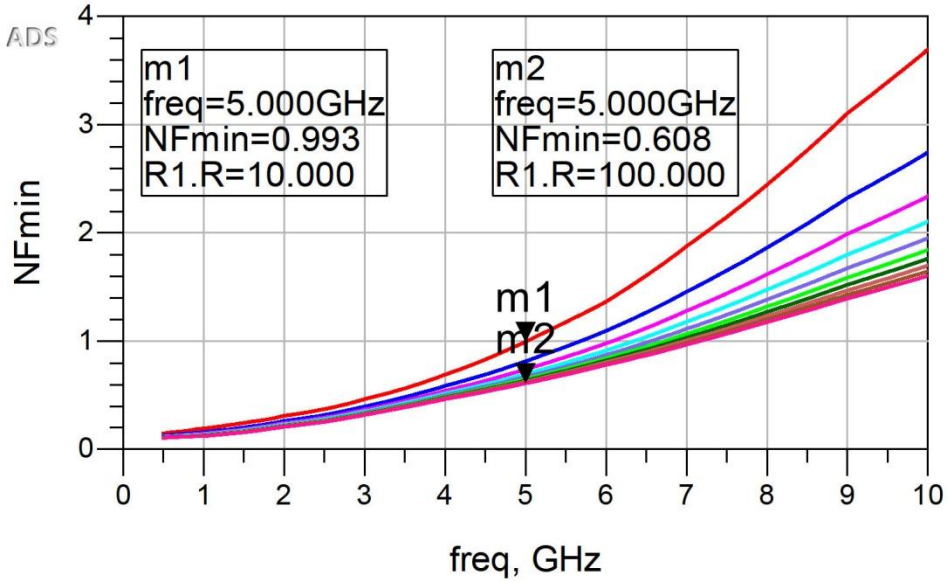


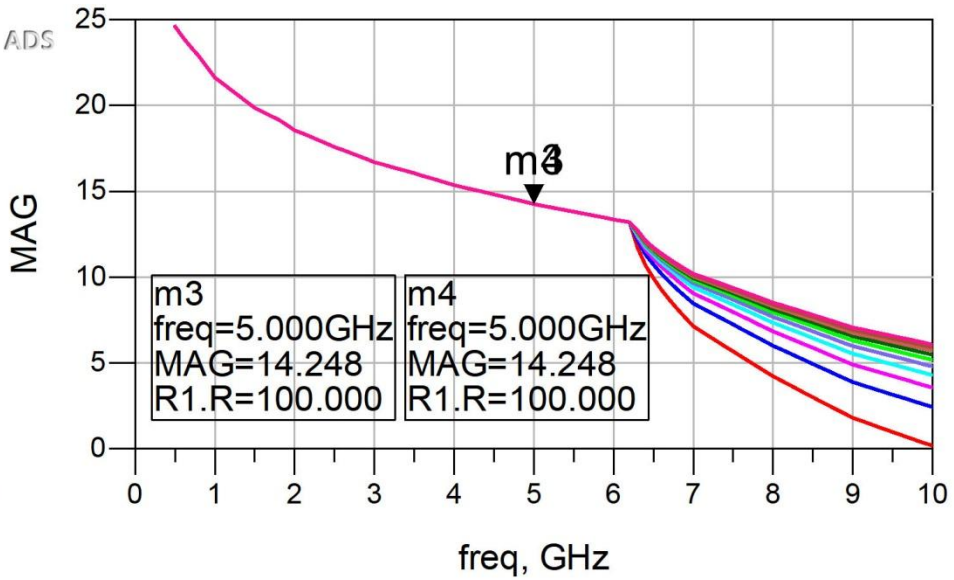


$$R_{SL} = 1 \div 10\ \Omega$$

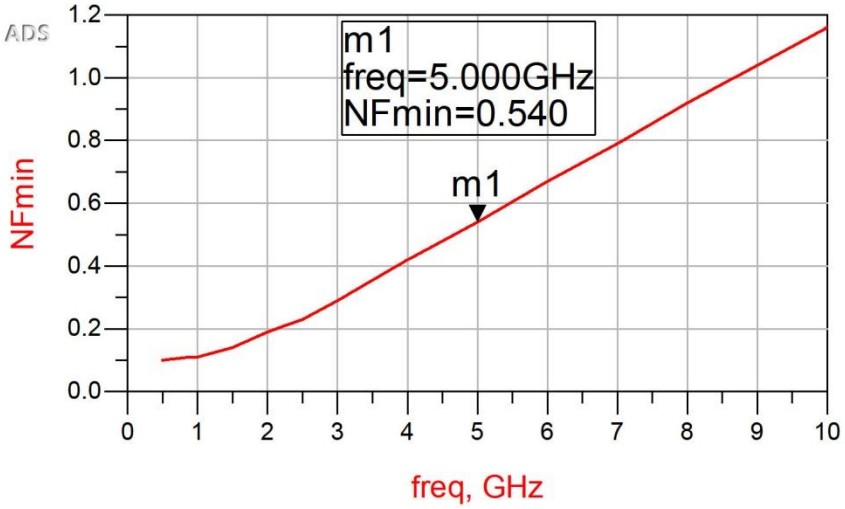


Stabilization, output shunt resistor





$$R_{PL} = 10 \div 100\ \Omega$$



Noise figure of a two-port amplifier

- 3 noise parameters (2reals + 1 complex):

$$F_{\min}, r_n = \frac{R_N}{Z_0}, \Gamma_{opt}$$

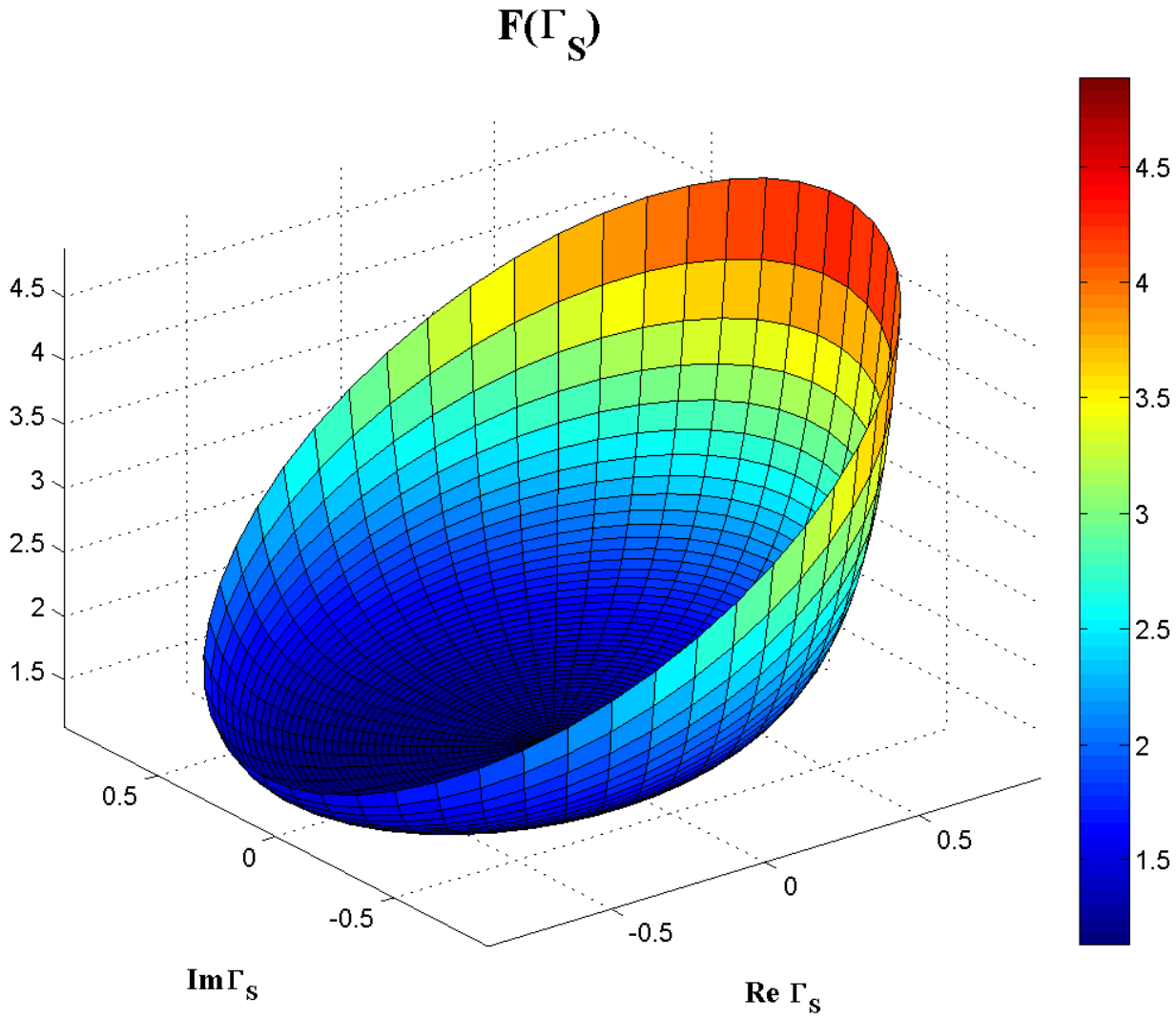
$$F = F_{\min} + \frac{R_N}{G_S} \cdot \left| Y_S - Y_{opt} \right|^2 \qquad Y_S = \frac{1}{Z_0} \cdot \frac{1-\Gamma_S}{1+\Gamma_S} \qquad Y_{opt} = \frac{1}{Z_0} \cdot \frac{1-\Gamma_{opt}}{1+\Gamma_{opt}}$$

$$F = F_{\min} + 4 \cdot r_n \cdot \frac{\left| \Gamma_S - \Gamma_{opt} \right|^2}{\left(1 - \left| \Gamma_S \right|^2 \right) \cdot \left| 1 + \Gamma_{opt} \right|^2}$$

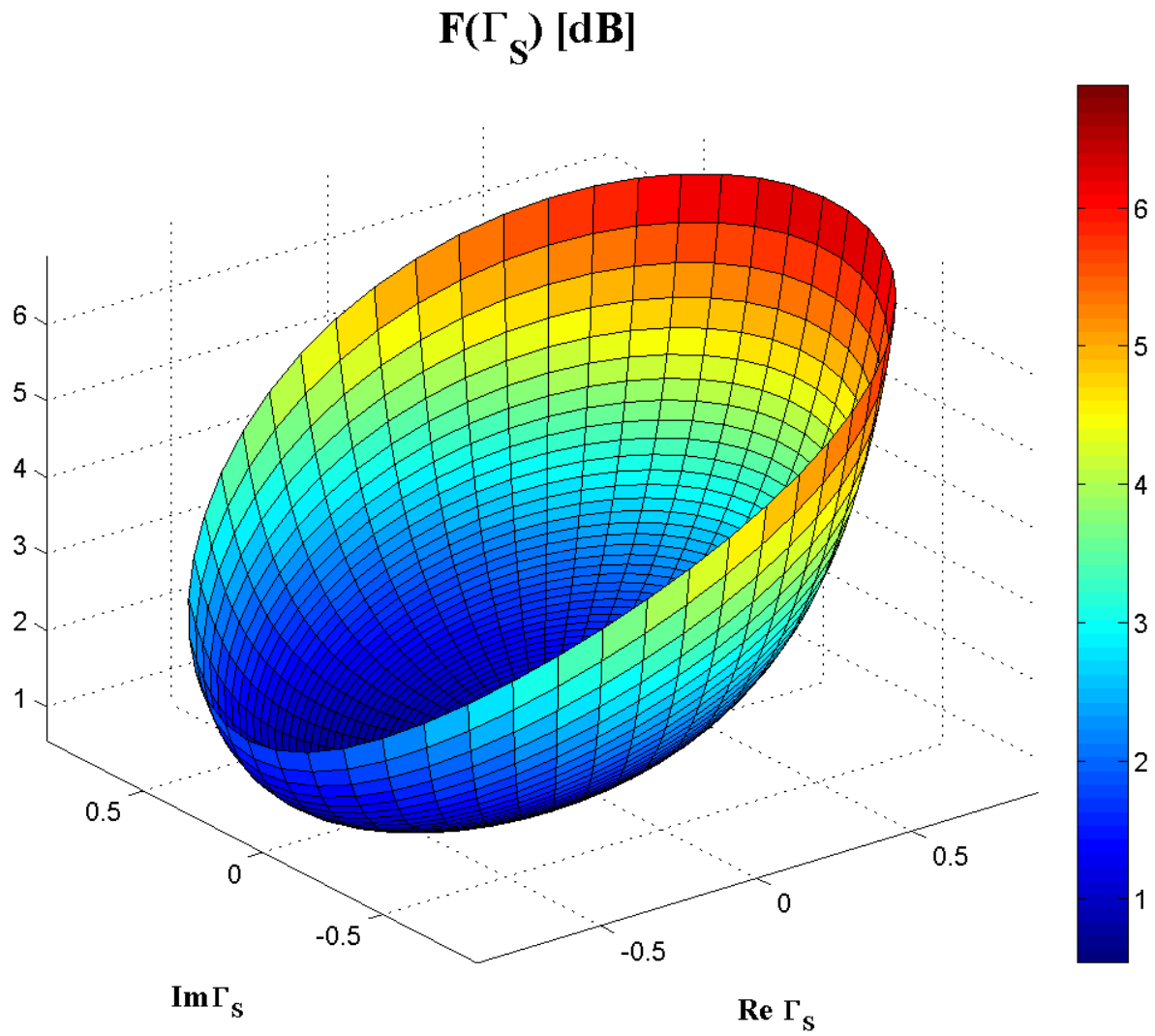
- Γ_{opt} optimum source reflection coefficient that results in minimum noise figure

$$\Gamma_S = \Gamma_{opt} \Rightarrow F = F_{\min}$$

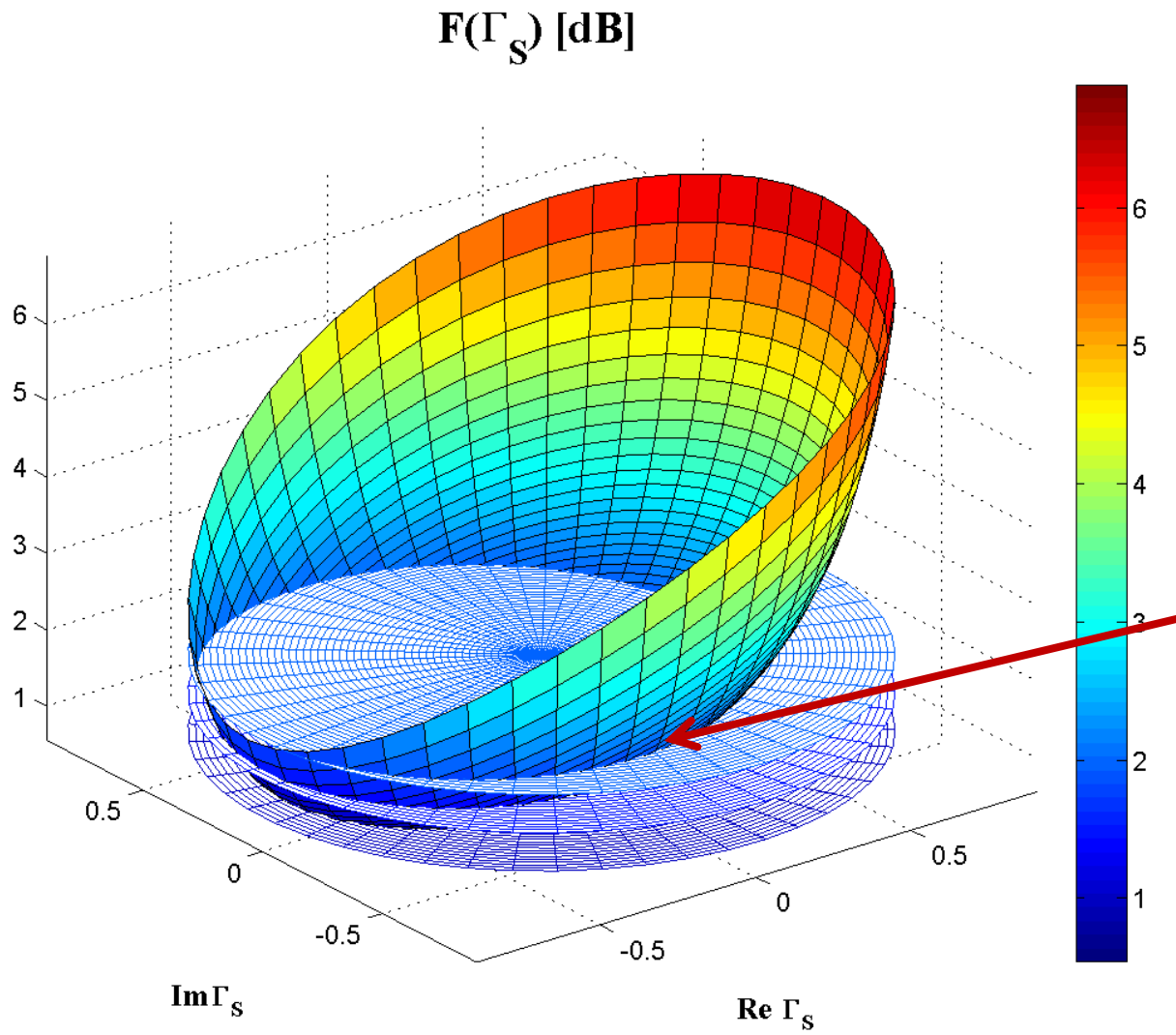
$F(\Gamma_s)$



$F[dB](\Gamma_s)$

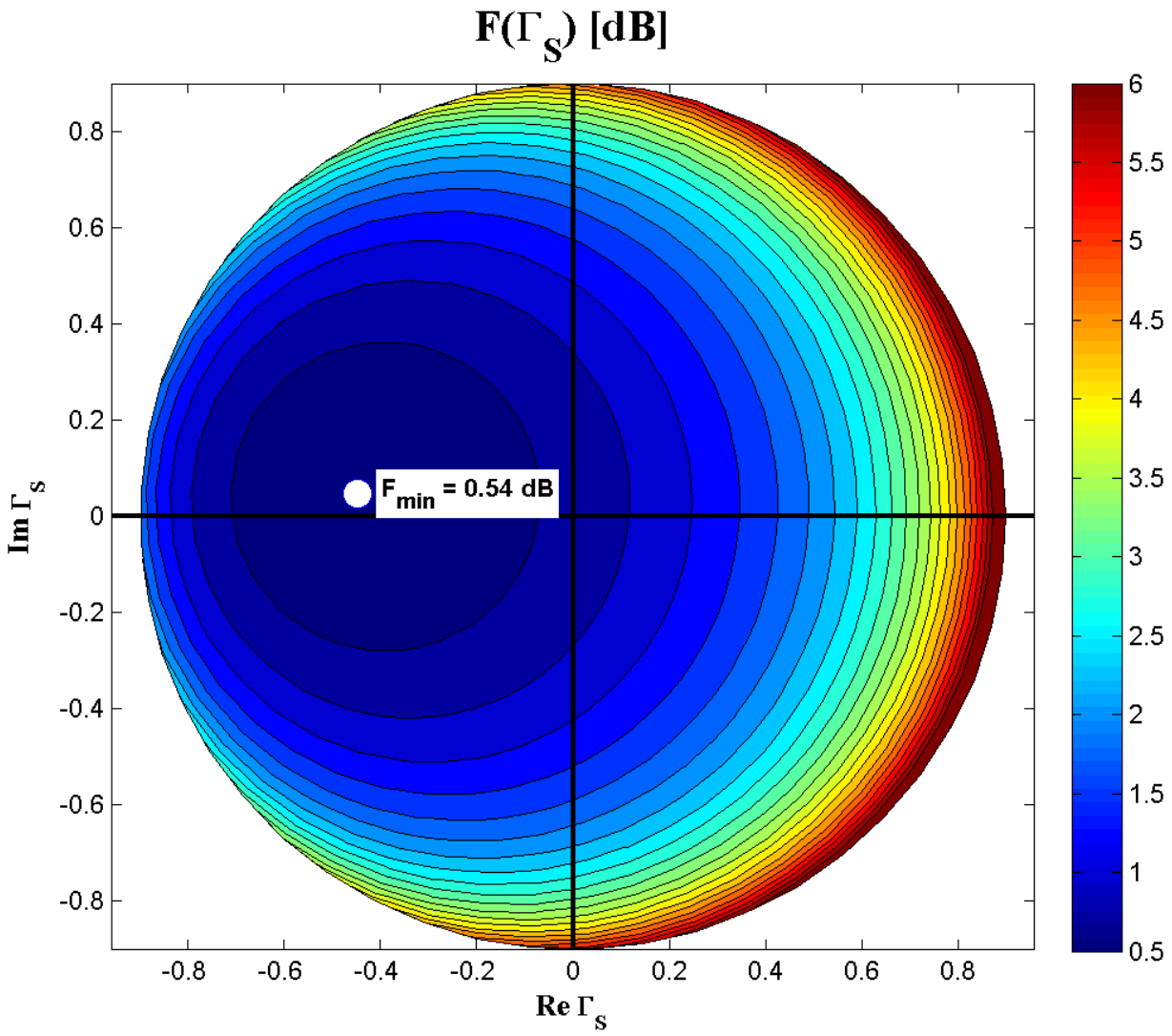


F[dB](Γ_S), constant value contours



Circles!!

$G_S[dB](\Gamma_S)$, constant value contours



Γopt = 0.45 ∠174°

Circles of constant noise figure

$$F = F_{\min} + 4 \cdot r_n \cdot \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{\left(1 - \left|\Gamma_S\right|^2\right) \cdot \left|1 + \Gamma_{opt}\right|^2}$$

- We define N (noise figure parameter)
  - **N** constant for **F** constant

$$N = \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{1 - \left|\Gamma_S\right|^2} = \frac{F - F_{\min}}{4 \cdot r_n} \cdot \left|1 + \Gamma_{opt}\right|^2$$

$$\left(\Gamma_S - \Gamma_{opt}\right) \cdot \left(\Gamma_S^* - \Gamma_{opt}^*\right) = N \cdot \left(1 - \left|\Gamma_S\right|^2\right)$$

$$\Gamma_S \cdot \Gamma_S^* + N \cdot \left|\Gamma_S\right|^2 - \left(\Gamma_S \cdot \Gamma_{opt}^* - \Gamma_S^* \cdot \Gamma_{opt}\right) + \Gamma_{opt} \cdot \Gamma_{opt}^* = N$$

$$\Gamma_S \cdot \Gamma_S^* - \frac{\Gamma_S \cdot \Gamma_{opt}^* - \Gamma_S^* \cdot \Gamma_{opt}}{N+1} + \Gamma_{opt} \cdot \Gamma_{opt}^* = \frac{N - \left|\Gamma_{opt}\right|^2}{N+1} + \frac{\left|\Gamma_{opt}\right|^2}{(N+1)^2}$$

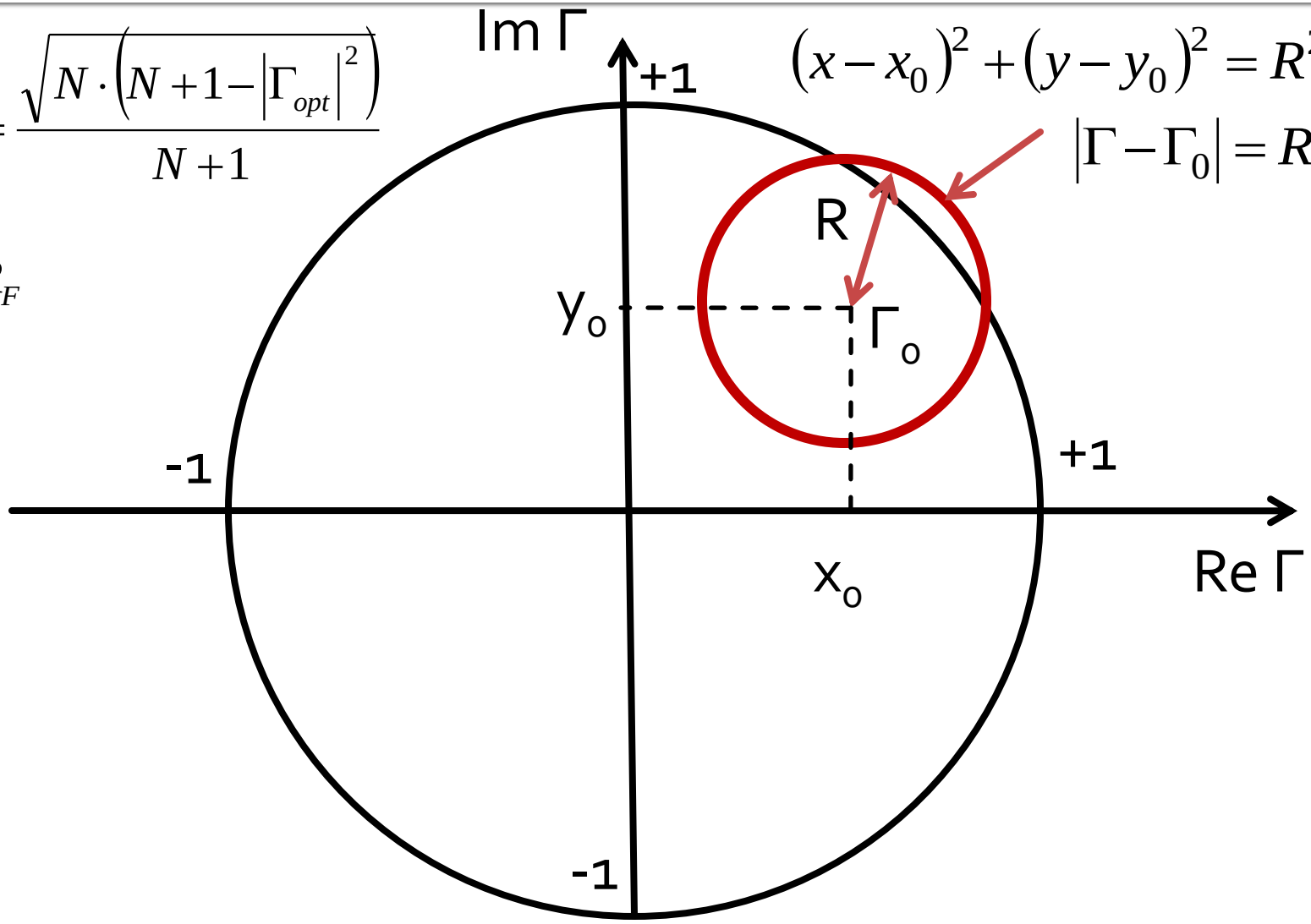
Circles of constant noise figure

$$\left|\Gamma_S - \frac{\Gamma_{opt}}{N+1}\right| = \frac{\sqrt{N \cdot \left(N+1-\left|\Gamma_{opt}\right|^2\right)}}{N+1}$$

$$\left|\Gamma_S - C_F\right| = R_F$$

$$(x-x_0)^2 + (y-y_0)^2 = R^2$$

$$\left|\Gamma - \Gamma_0\right| = R$$



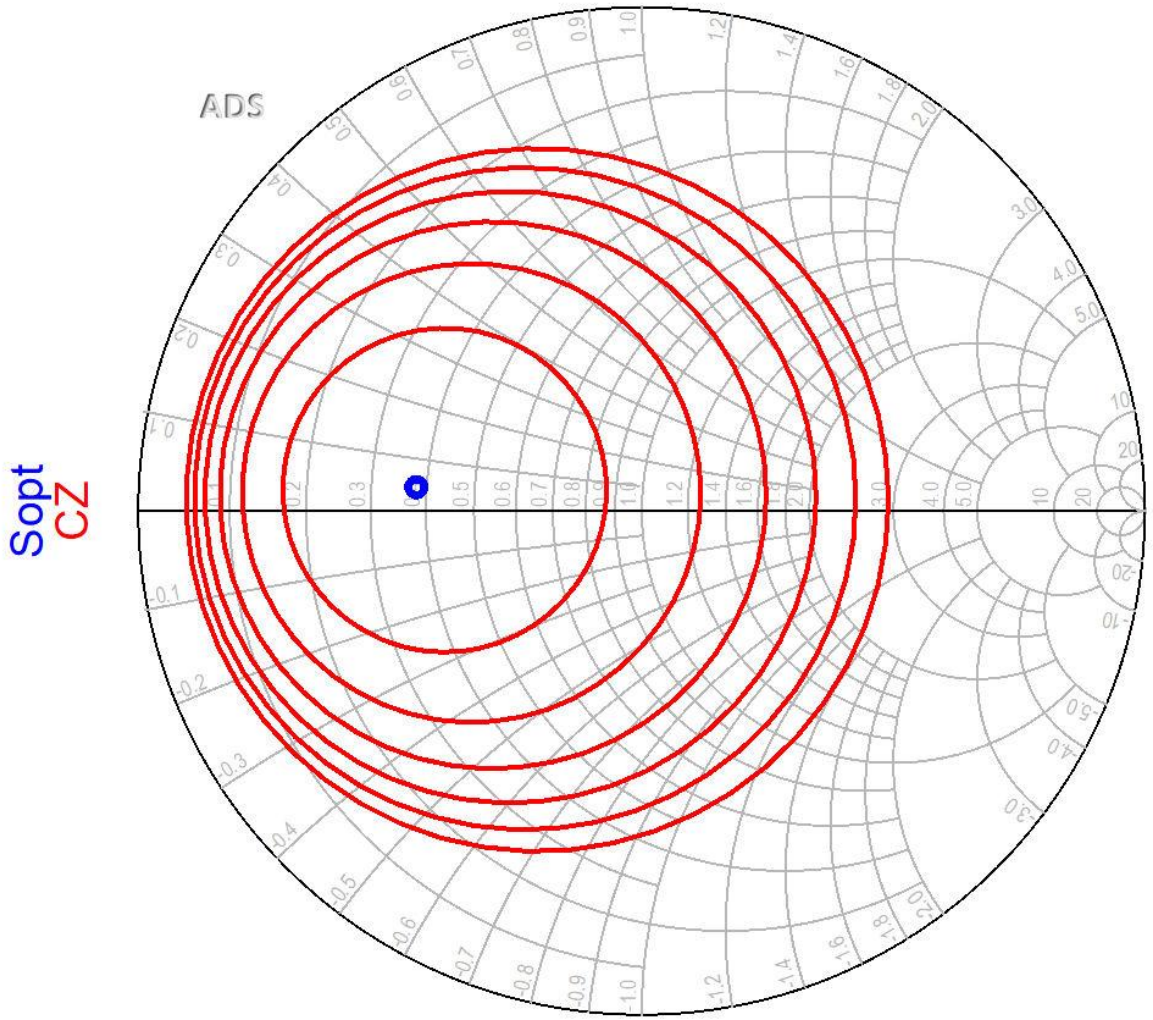
Circles of constant noise figure

$$\left|\Gamma_S - \frac{\Gamma_{opt}}{N+1}\right| = \frac{\sqrt{N\cdot\left(N+1-\left|\Gamma_{opt}\right|^2\right)}}{N+1} \qquad N = \frac{F-F_{\min}}{4\cdot r_n}\cdot\left|1+\Gamma_{opt}\right|^2$$

$$\left|\Gamma_S - C_F\right| = R_F \qquad C_F = \frac{\Gamma_{opt}}{N+1} \qquad R_F = \frac{\sqrt{N\cdot\left(N+1-\left|\Gamma_{opt}\right|^2\right)}}{N+1}$$

- The locus in the complex plane Γ_S of the points with constant noise figure is a circle
- **Interpretation**: Any reflection coefficient Γ_S which plotted in the complex plane lies **on** the circle drawn for F_{circle} will lead to a noise factor $F = F_{circle}$
  - Any reflection coefficient Γ_S plotted **outside** this circle will lead to a noise factor $F > F_{circle}$
  - Any reflection coefficient Γ_S plotted **inside** this circle will lead to a noise factor $F < F_{circle}$

ADS

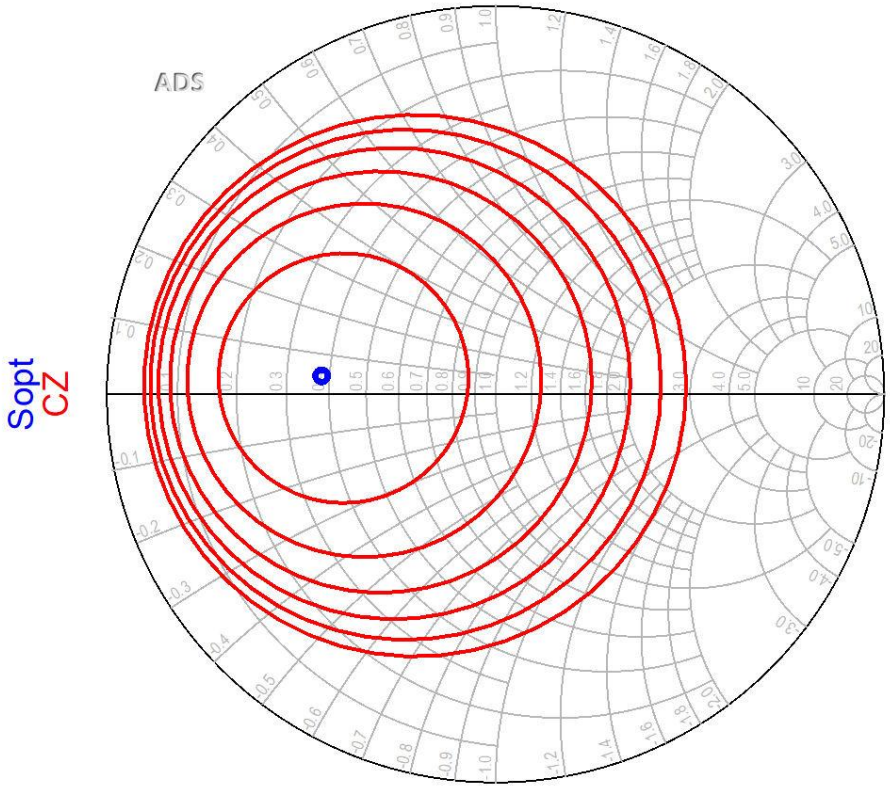


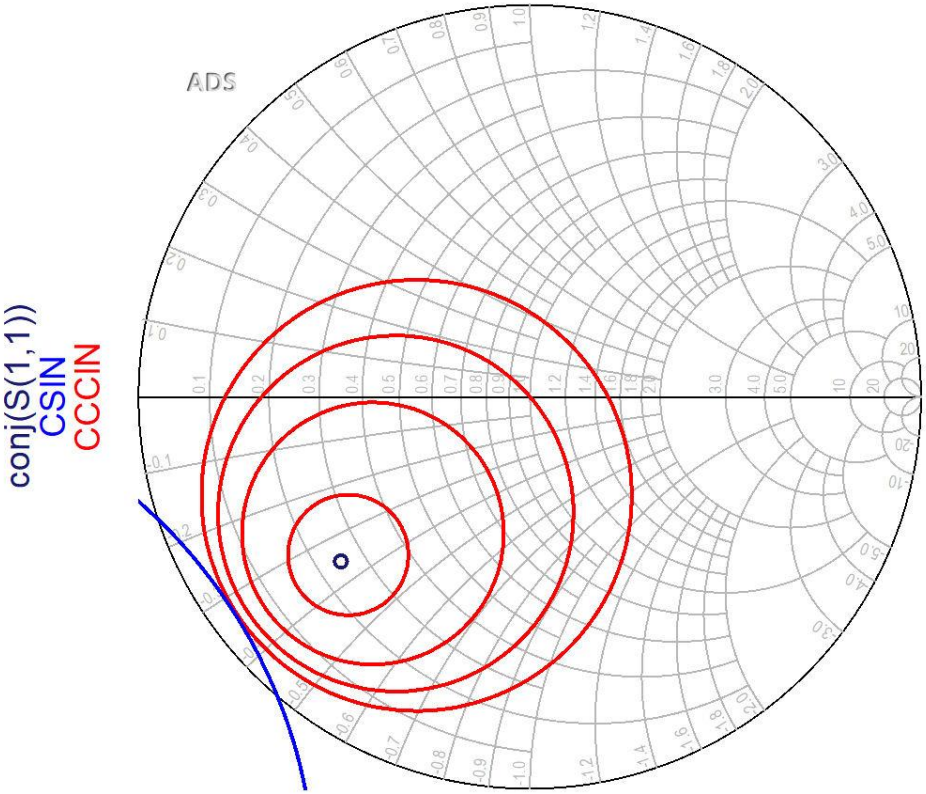
Circles of constant noise figure

- The noise internally generated by the transistor depends **only** by the input matching circuit
- A minimum noise figure is possible (NF_{min} – a datasheet/"s2p file" parameter for the transistor)
- If we design a low noise amplifier (**LNA**) the usual design technique is as follows:
  - design of the input matching circuit solely (largely) for noise optimization
  - design of output matching circuit for gain compensation/optimization (if lossy circuits are used the output matching circuit noise can be added but the transistor noise is not influenced)

LNA – Low Noise Amplifier

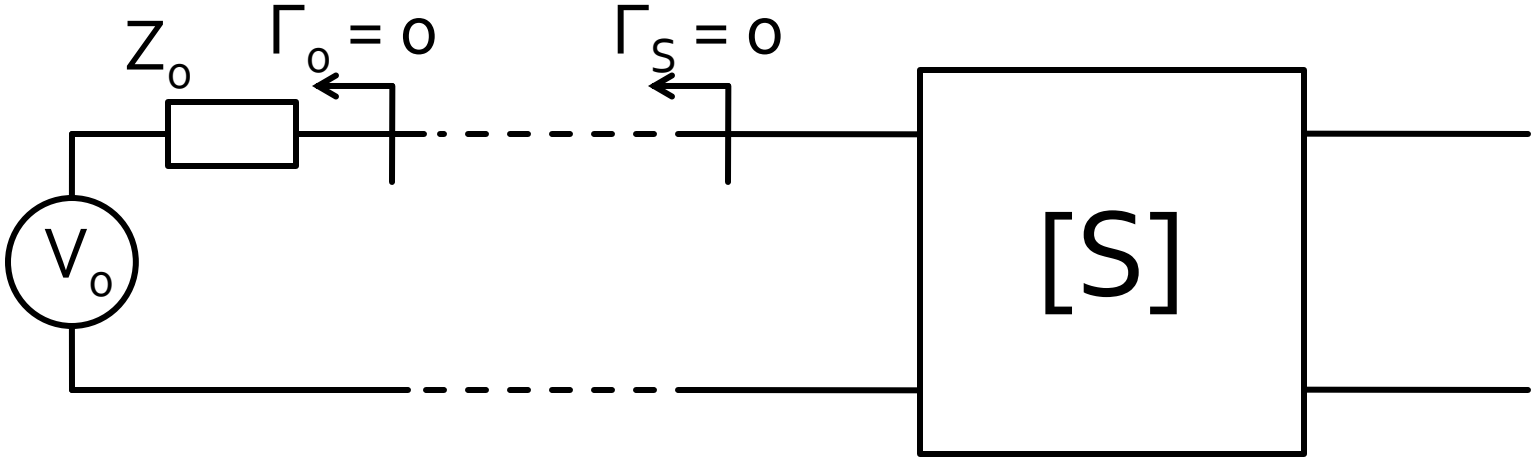
- Usually a transistor suitable for implementing an LNA at a certain frequency will have input gain circles and noise circles in the same area for Γ_S





Matching – 1

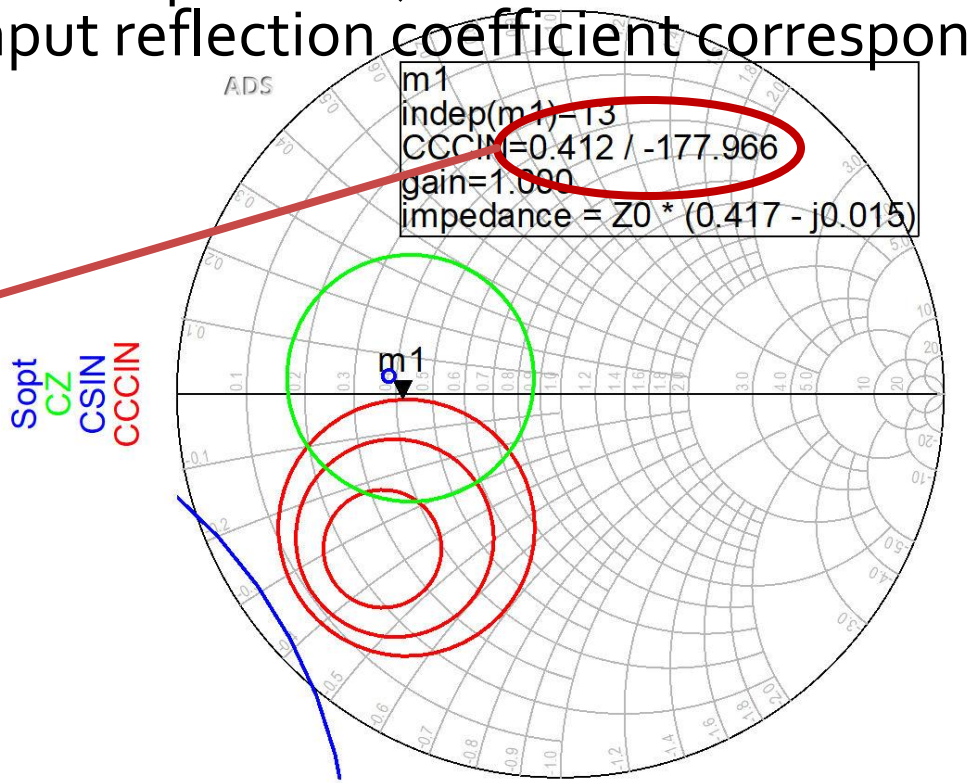
- Connecting the amplifier (transistor) directly to the source with Zo generate a reflection coefficient seen towards the source equal with **o** (complex number, $\Gamma_o = o + o \cdot j$)
  - most of the time this reflection coefficient does not offer optimum noise/gain



Matching – 2

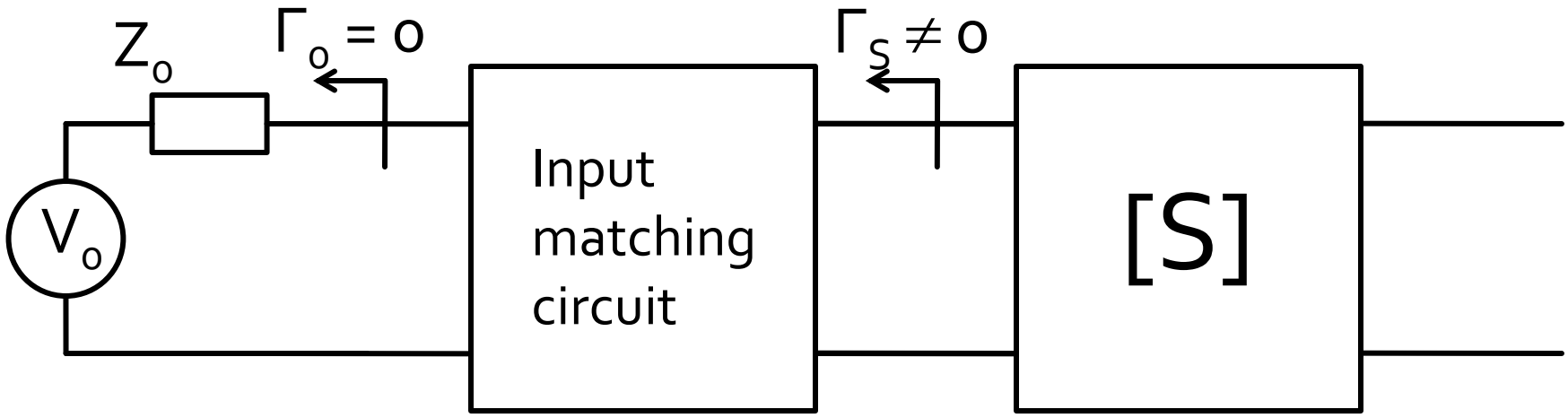
- We plot on the complex plane (Smith Chart) the stability/gain/noise circles (depending on the particular application)
- We choose a point with a suitable position relative to these circles (also application dependent)
- We determine the input reflection coefficient corresponding to this point, Γ_S

$$\Gamma_S = 0.412 \angle -177.966^\circ$$



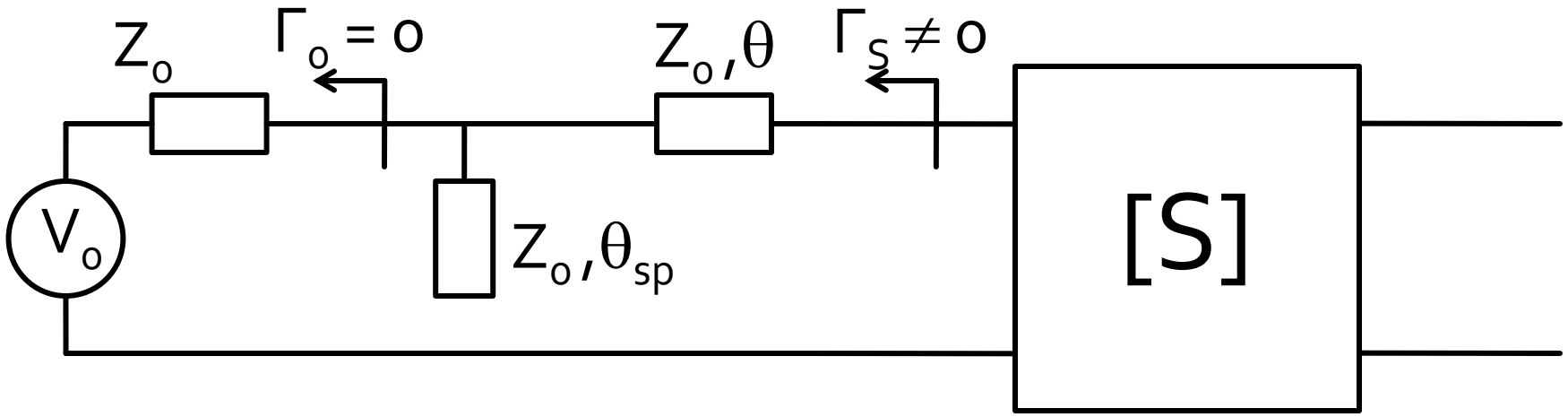
Matching – 3

- We insert the input matching circuits which allows the transistor to see towards the source the previously determined reflection coefficient Γ_S



Matching – 4

- Easiest to design matching section consists in the insertion of (in order from the transistor towards the Z_o source):
  - a series Z_o line, with electrical length θ
  - a shunt stub, open-circuited, made from a Z_o line, with electrical length θ_{sp}



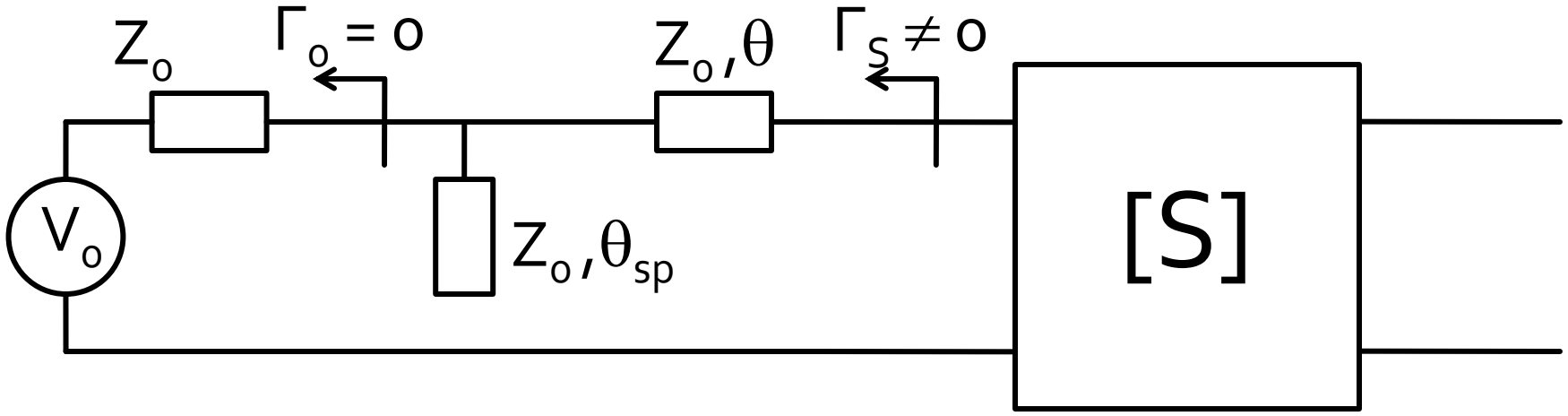
Matching – 5

- Computation depends solely on Γ_S (magnitude and phase)

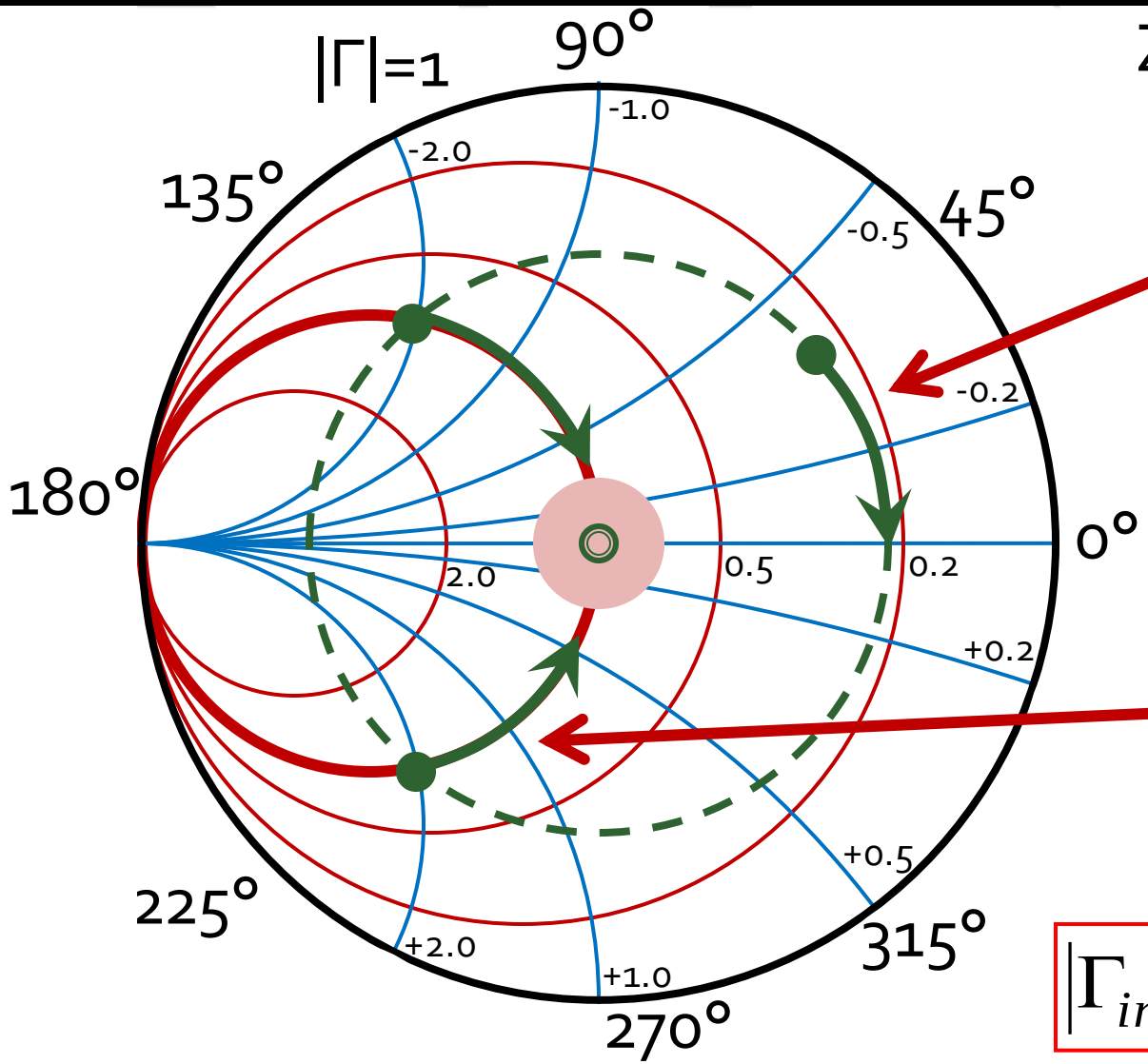
$$\cos(\varphi_S + 2\theta) = -|\Gamma_S|$$

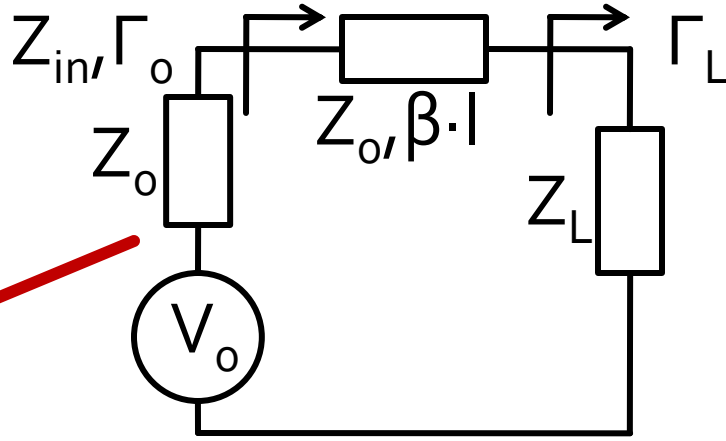
$$\tan\theta_{sp} = \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1-|\Gamma_S|^2}}$$

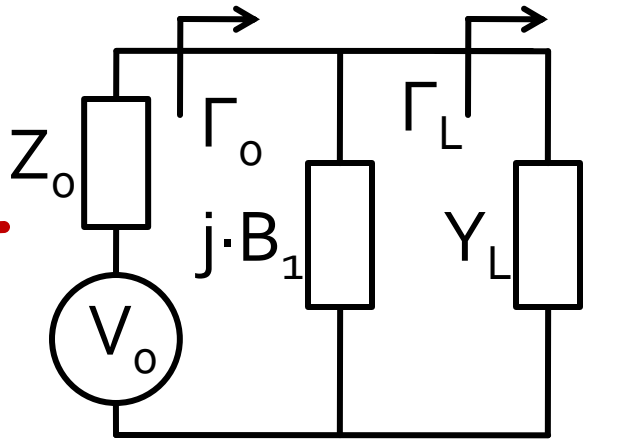
- The sign (+/-) chosen for the series line equation imposes the sign used for the shunt stub equation



Shunt stub matching, L7







$$|\Gamma_{in}| = |\Gamma_L|$$

$$g_{in} = 1$$

Example, LNA @ 5 GHz

- ATF-34143 **at Vds=3V Id=20mA.**
- @5GHz
  - S11 = 0.64∠139°
  - S12 = 0.119∠-21°
  - S21 = 3.165 ∠16°
  - S22 = 0.22 ∠146°
  - Fmin = 0.54 (**tipic [dB]**
  - Γ_{opt} = 0.45 ∠174°
  - r_n = 0.03

```
!ATF-34143
!S-PARAMETERS at Vds=3V  Id=20mA.   LAST UPDATED 01-29-99

# ghz s ma r 50

2.0  0.75  -126  6.306  90  0.088  23  0.26  -120
2.5  0.72  -145  5.438  75  0.095  15  0.25  -140
3.0  0.69  -162  4.762  62  0.102  7  0.23  -156
4.0  0.65  166  3.806  38  0.111  -8  0.22  174
5.0  0.64  139  3.165  16  0.119  -21  0.22  146
6.0  0.65  114  2.706  -5  0.125  -35  0.23  118
7.0  0.66  89  2.326  -27  0.129  -49  0.25  91
8.0  0.69  67  2.017  -47  0.133  -62  0.29  67
9.0  0.72  48  1.758  -66  0.135  -75  0.34  46

!FREQ   Fopt   GAMMA OPT      RN/Zo
!GHZ    dB     MAG    ANG     -

2.0  0.19  0.71  66  0.09
2.5  0.23  0.65  83  0.07
3.0  0.29  0.59  102  0.06
4.0  0.42  0.51  138  0.03
5.0  0.54  0.45  174  0.03
6.0  0.67  0.42  -151  0.05
7.0  0.79  0.42  -118  0.10
8.0  0.92  0.45  -88  0.18
9.0  1.04  0.51  -63  0.30
10.0  1.16  0.61  -43  0.46
```

Example, LNA @ 5 GHz

- Low Noise Amplifier
- At the input matching a compromise is required between:
  - noise (~~input~~ constant noise figure circles)
  - gain (input constant gain circles)
  - stability (input stability circle)
- At the output matching noise **is not influenced**. A compromise is required between :
  - gain (output constant gain circles)
  - stability (output stability circle)

Example, LNA @ 5 GHz

$$U = \frac{|S_{12}| \cdot |S_{21}| \cdot |S_{11}| \cdot |S_{22}|}{\left(1-|S_{11}|^2\right) \cdot \left(1-|S_{22}|^2\right)} = 0.094$$

$$-0.783\ dB < G_T[dB] - G_{TU}[dB] < 0.861\ dB$$

$$G_{TU\,\max} = \frac{1}{1-|S_{11}|^2} \cdot |S_{21}|^2 \cdot \frac{1}{1-|S_{22}|^2} = 17.83$$

$$G_{TU\,\max}[dB] = 12.511\ dB$$

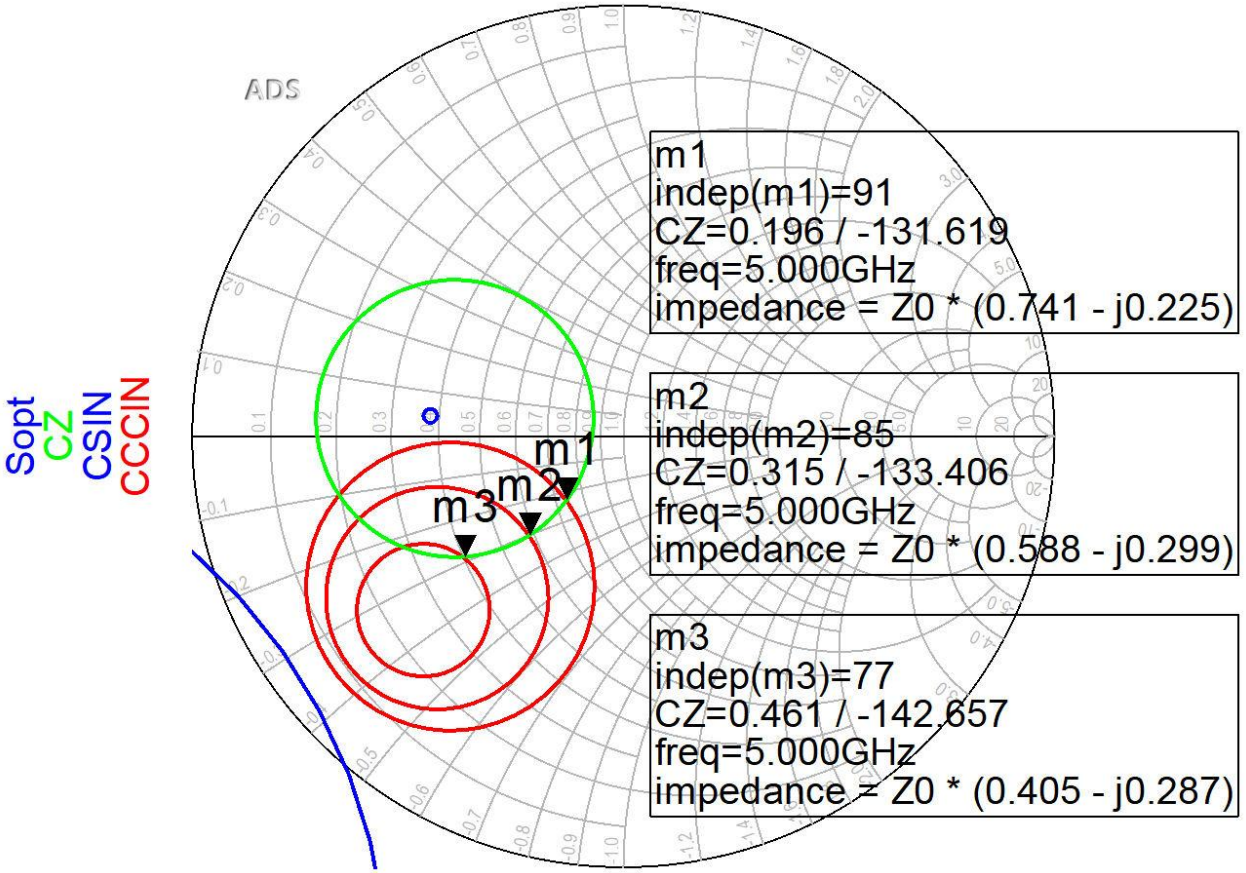
$$G_0 = |S_{21}|^2 = 10.017 = 10.007\ dB$$

$$G_{S\,\max} = \frac{1}{1-|S_{11}|^2} = 1.694 = 2.289\ dB$$

$$G_{L\,\max} = \frac{1}{1-|S_{22}|^2} = 1.051 = 0.215\ dB$$

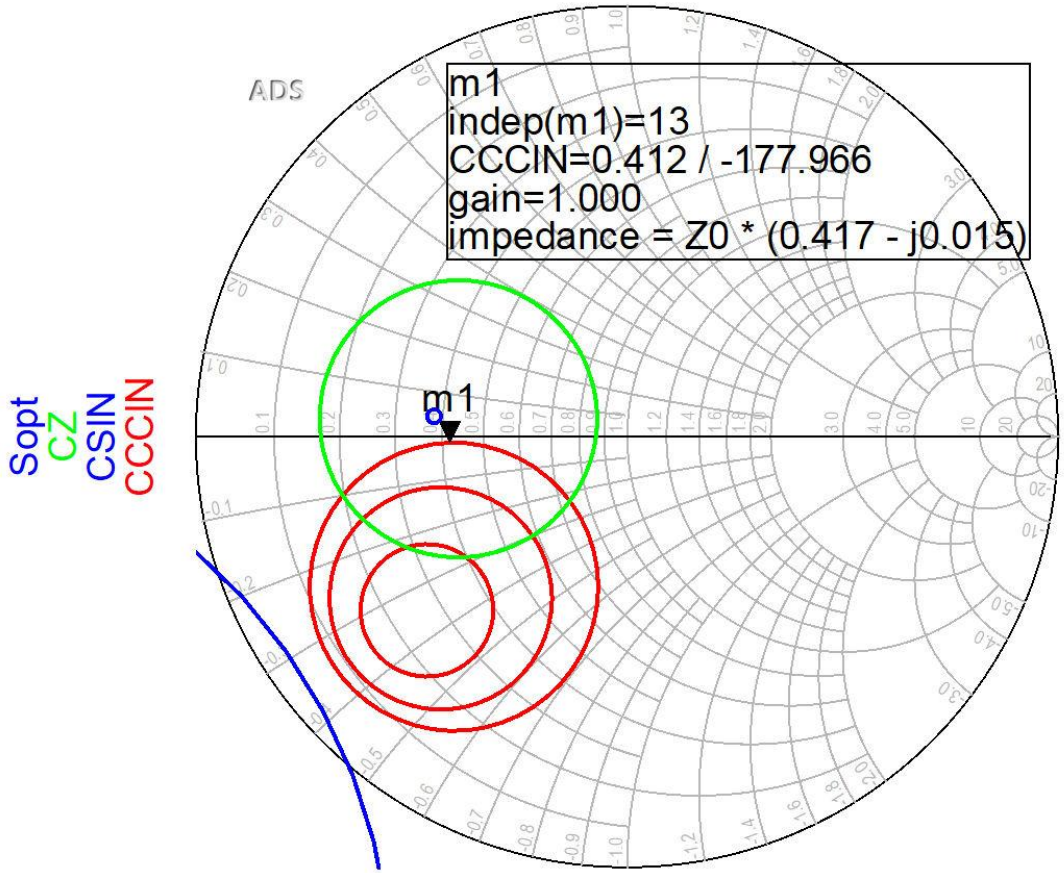
- In this particular case G_{Lmax} = 0.21 dB, the transistor could be used directly connected to the 50Ω load
- The absence of the output matching circuit **is not** recommended. While the attainable power gain is low, it's absence eliminates the possibility to use it to compensate an improper gain generated by the noise optimization of the input matching circuit

Input matching circuit



- For the input matching circuit
  - noise circle CZ: 0.75dB
  - input constant gain circles CCCIN: 1dB, 1.5dB, 2 dB
- We choose (small Q → wide bandwidth) position m1

Input matching circuit



- If we can afford a 1.2dB decrease of the input gain for better NF,Q (Gs = 1 dB), position m1 above is better
- We obtain better (smaller) NF

Input matching circuit

- Position m1 in complex plane (Smith Chart)

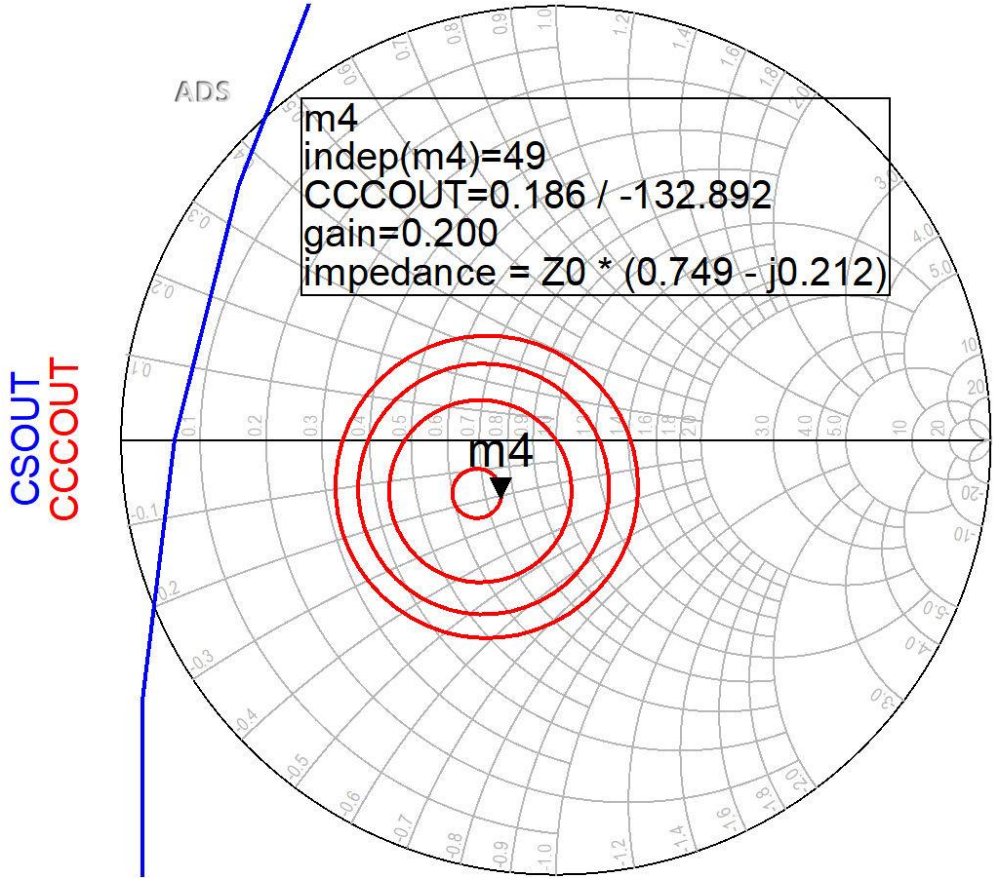
$$\Gamma_S = 0.412 \angle -178^\circ \qquad |\Gamma_S| = 0.412; \quad \varphi = -178^\circ$$

$$\cos(\varphi + 2\theta) = -|\Gamma_S| \qquad \mathrm{Im}[y_S(\theta)] = \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\cos(\varphi + 2\theta) = -0.412 \Rightarrow \qquad (\varphi + 2\theta) = \pm 114.33^\circ$$

$$(\varphi + 2\theta) = \begin{cases} +114.33^\circ \\ -114.33^\circ \end{cases} \quad \theta = \begin{cases} 146.2^\circ \\ 31.8^\circ \end{cases} \quad \mathrm{Im}[y_S(\theta)] = \begin{cases} -0.904 \\ +0.904 \end{cases} \quad \theta_{sp} = \begin{cases} 137.9^\circ \\ 42.1^\circ \end{cases}$$

Output matching circuit



- output constant gain circles CCCOUT: -0.4dB, -0.2dB, 0dB, +0.2dB
- the lack of noise restrictions allows optimization for better gain (close to maximum – position m4)

Output matching circuit

- Position m4 in complex plane (Smith Chart)

$$\Gamma_L = 0.186 \angle -132.9^\circ \qquad |\Gamma_L| = 0.186; \quad \varphi = -132.9^\circ$$

$$\cos(\varphi + 2\theta) = -|\Gamma_L| \qquad \mathrm{Im}[y_L(\theta)] = \frac{-2 \cdot |\Gamma_L|}{\sqrt{1 - |\Gamma_L|^2}} = -0.379$$

$$\cos(\varphi + 2\theta) = -0.186 \Rightarrow \quad (\varphi + 2\theta) = \pm 100.72^\circ$$

$$(\varphi + 2\theta) = \begin{cases} +100.72^\circ \\ -100.72^\circ \end{cases} \quad \theta = \begin{cases} 116.8^\circ \\ 16.1^\circ \end{cases} \quad \mathrm{Im}[y_L(\theta)] = \begin{cases} -0.379 \\ +0.379 \end{cases} \quad \theta_{sp} = \begin{cases} 159.3^\circ \\ 20.7^\circ \end{cases}$$

LNA

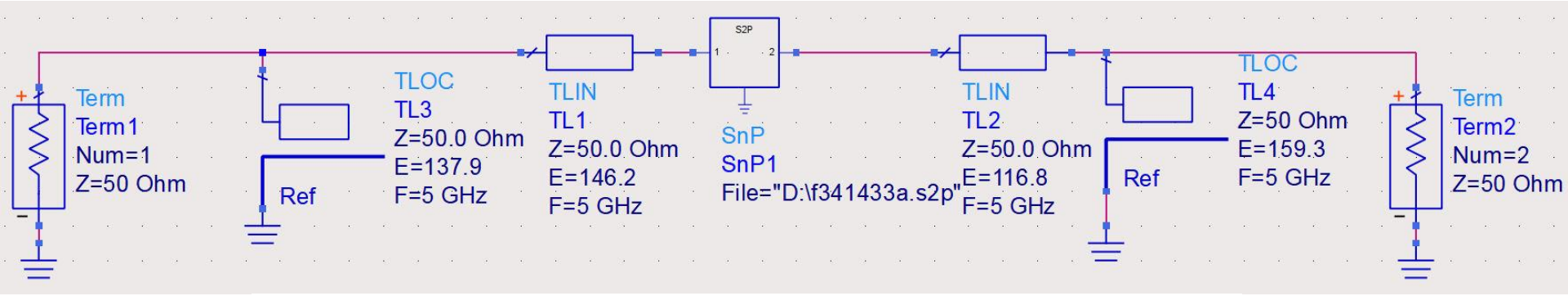
- We estimate a gain (in unilateral assumption, ±0.9 dB)

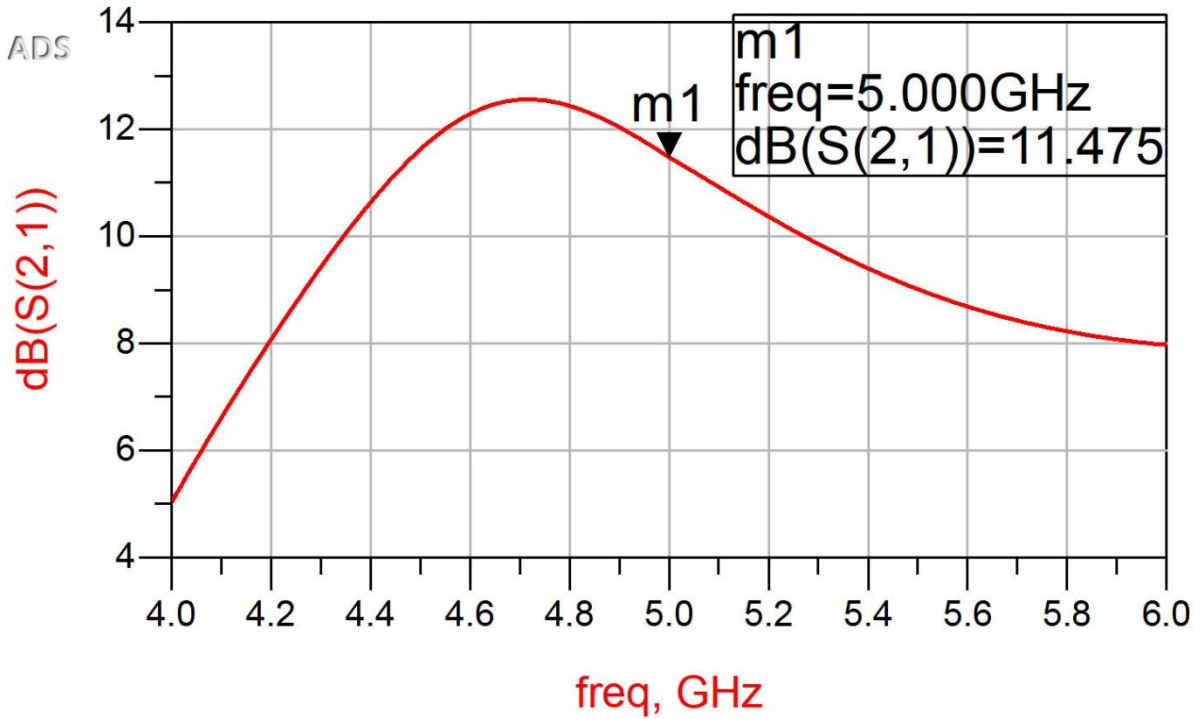
$$G_T[dB] = G_S[dB] + G_0[dB] + G_L[dB]$$

$$G_T[dB] = 1\,dB + 10\,dB + 0.2\,dB = 11.2\,dB$$

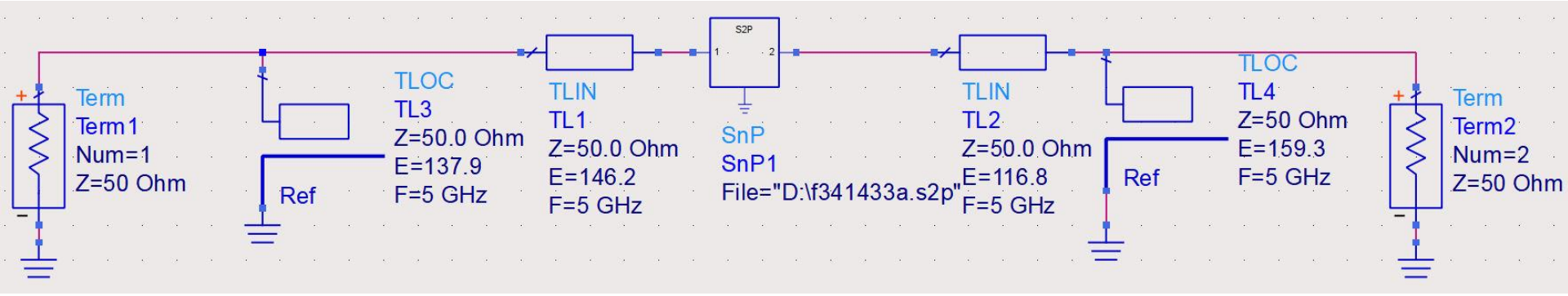
- We estimate a noise factor well bellow 0.75dB (quite close to the minimum ~0.6 dB)

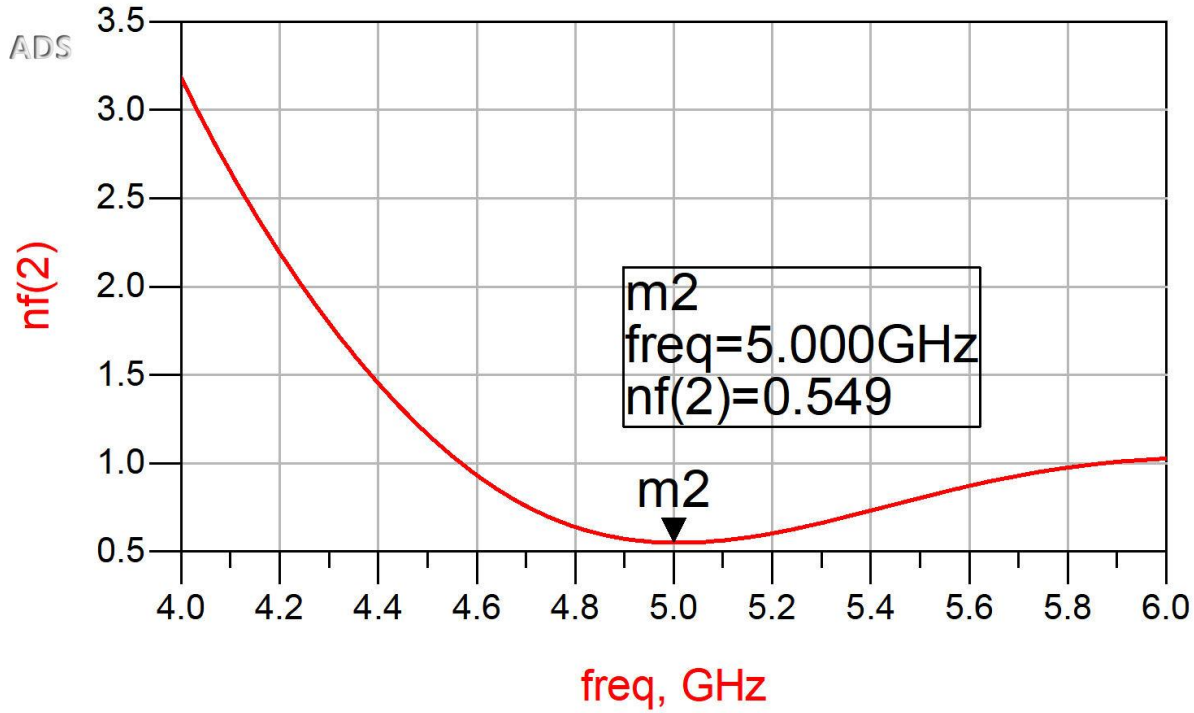
ADS


Term
Term1
Num=1
Z=50 Ohm
Ref
TLOC
TL3
Z=50.0 Ohm
E=137.9
F=5 GHz
TLIN
TL1
Z=50.0 Ohm
E=146.2
F=5 GHz
S2P
1
2
SnP
SnP1
File="D:\f341433a.s2p"
TLIN
TL2
Z=50.0 Ohm
E=116.8
F=5 GHz
Ref
TLOC
TL4
Z=50 Ohm
E=159.3
F=5 GHz
Term
Term2
Num=2
Z=50 Ohm


ADS
m1
freq=5.000GHz
dB(S(2,1))=11.475
dB(S(2,1))
14
12
10
8
6
4
4.0
4.2
4.4
4.6
4.8
5.0
5.2
5.4
5.6
5.8
6.0
freq, GHz

ADS


Term
Term1
Num=1
Z=50 Ohm
Ref
TLOC
TL3
Z=50.0 Ohm
E=137.9
F=5 GHz
TLIN
TL1
Z=50.0 Ohm
E=146.2
F=5 GHz
S2P
1
2
SnP
SnP1
File="D:\f341433a.s2p"
TLIN
TL2
Z=50.0 Ohm
E=116.8
F=5 GHz
Ref
TLOC
TL4
Z=50 Ohm
E=159.3
F=5 GHz
Term
Term2
Num=2
Z=50 Ohm


ADS
nf(2)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
4.0
4.2
4.4
4.6
4.8
5.0
5.2
5.4
5.6
5.8
6.0
m2
freq=5.000GHz
nf(2)=0.549
m2
freq, GHz

Contact

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